22. ANALYSIS OF DOUBLE STAR RESULTS

A solution for the relative astrometry and photometry of double stars has been obtained from the Hipparcos observations, strictly based on the methods introduced in Chapter 13 of this volume. Although the Hipparcos processing provides an indication of the internal error, a better evaluation of the true external error can only be obtained by comparison with ground-based observations of comparable accuracy. We discuss in this chapter two such comparisons: the first analyses the results of the relative astrometry with respect to the best ground-based observations by speckle interferometry, for about 1000 stars common to Hipparcos and to the CHARA programme; the second investigates the photometric solution in relation to the CCD photometric observations carried out at La Palma over a sample of similar size common to both programmes.

22.1. Introduction

The details of the methods implemented by FAST and NDAC to determine the astrometry and photometry of double and multiple stars are given in Chapter 13 of this volume, along with the main properties of the solution. The precision for the relative astrometry (separation ϱ and position angle θ) was shown to be mainly dependent on the magnitude difference but not very sensitive to the separation, at least for separations less than 15 arcsec. A fit of the median of the standard error of the separation yields the following useful formula for the precision as a function of the magnitude difference Δm:

$$\log \sigma_\varrho \simeq \max(0.75,\ 0.5 + 0.3\Delta m) \qquad [22.1]$$

where the standard error σ_ϱ is expressed in milliarcsec (Figure 22.1). For the subset of easy double stars, with $\varrho \gtrsim 0.2$ arcsec and $\Delta m \lesssim 2$ mag, the separation could be obtained with a precision better than 10 mas and in many instances than 5 mas. For relative photometry, Hipparcos provides the best homogeneous and full-sky coverage for a sample of 12 000 systems with a precision of a few 0.01 mag. The precision of the magnitude difference is, again, primarily dependent upon the magnitude difference itself and to a lesser extent on the separation. For separations larger than 0.3 arcsec a smooth representation of the median of the standard error is given by:

$$\log \sigma_{\Delta m} \simeq -1.7 + 0.25\Delta m \qquad [22.2]$$

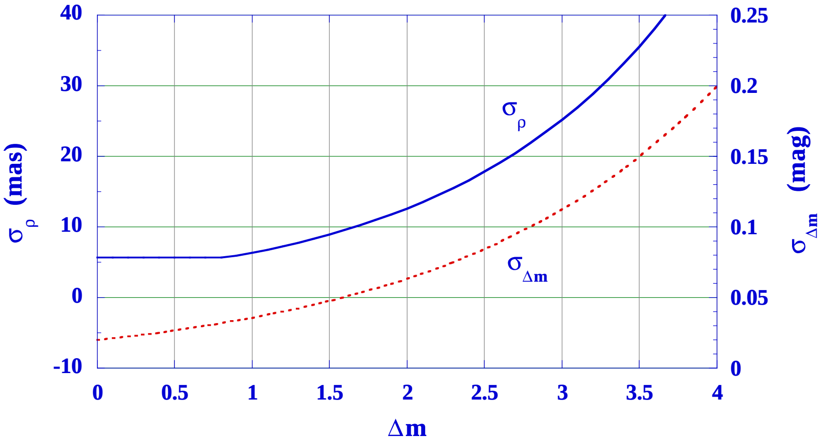

Figure 22.1. *Average standard errors of the relative astrometric (left scale) and photometric (right scale) solution of the binary stars as a function of the magnitude difference.*

(see Figure 22.1). For smaller separations the typical standard error in Δm grows sharply with $1/\varrho$ and the magnitude difference and separation become strongly correlated.

It is difficult to assess to what extent these internal errors are representative of the true errors, despite the effort of the data analysis groups to provide as realistic an evaluation of this error as possible. In addition, systematic errors both in astrometry and photometry are possible and very likely to exist especially at the two extremes: small separations ($\varrho < 0.15$ arcsec) and large magnitude differences ($\Delta m > 3$ mag). The analyses presented in this chapter attempt to provide a more objective assessment of these errors through the results of a comparison of the Hipparcos results with the best ground-based data to date, the speckle astrometric measurements and the CCD photometric observations.

22.2. Relative Astrometry

Ground-Based Material

The only sizable set of observations of relative astrometry of multiple systems matching the quality of the Hipparcos data is provided by the speckle observations and occultation timings compiled in the various versions of the CHARA Catalogue. The following work is based on Version 3, available on the World Wide Web (Hartkopf *et al.* 1996). Preliminary comparisons were carried out on a small sample during the Hipparcos data reduction (Mignard *et al.* 1995) and based on a previous version of the CHARA Catalogue or on data published in the Astronomical Journal, and led to the evidence of a small bias in separation between Hipparcos and the ground-based observations for separations above 0.6 arcsec. All these observations are included in the present analysis.

To be more precise, the third CHARA Catalogue includes all measures of binary and multiple star systems obtained by modern high-resolution techniques (speckle interferometry, photoelectric occultation timings) as well as negative examination for duplicity, as of December 1995. For each observation reported, there is an indication of the observer and the method employed. Each system is identified by one or several of the following identifiers: ADS, HR, HD, SAO, WDS. The observation records give the date of observation, position angle, separation (or upper limit when no detection was possible), and, in some cases, an indication of the error. When available, the magnitude of each component is also given, but these data were not used for the present analysis because they are too scarce and have a relatively low accuracy compared to the CCD observations made at La Palma (Section 22.3).

Cross-Identifications

The Hipparcos identification number is obviously not among the various identifiers found in the CHARA Catalogue and it must be searched for by cross-identification. Unfortunately, none of the above identifiers is alone sufficient to find all the Hipparcos stars which are in the CHARA Catalogue. In principle the WDS identifier, available for every entry, should allow the correct identification of all the systems to be found by using properly truncated J2000 coordinates or the CCDM number when the system is known to be double in the Hipparcos Catalogue. While easy to implement, this method is not very reliable, yielding too many wrong identifications simply because of differences of one or two units in the last digit of rounded right ascension or declination between the WDS identifier and the Hipparcos truncated positions.

As a consequence systems clearly in the Hipparcos and CHARA Catalogues went unnoticed (i.e. were not recognised as Hipparcos objects) or were given a false Hipparcos identifier, only because the coordinates of two Hipparcos entries were a few minutes apart. (An example of such a situation arises with HIP 162 and HIP 171, which are different components of a wide system.) This method of identification was not used in a systematic way but only as a last resort after all the other identifiers had been exhausted. The other identifiers like HD or SAO do not suffer from this drawback, but unfortunately do not cover all the Hipparcos entries. Eventually all these possibilities were used in sequence and the final files were merged into a single one without redundant identifications.

There are 6280 entries in the CHARA Catalogue representing about 22 000 individual observations. Five thousand of these entries have a counterpart in the Hipparcos catalogue and 2100 are associated with at least one positive and reliable observation of separation and position angle. The number of systems with a double star solution in the Hipparcos catalogue is obviously smaller, of the order of 1700. Accurate numbers are given in the last column of Table 22.1. The 400 remaining systems are mainly close binaries and therefore not detected as non-single by Hipparcos.

Few systems were solved with more than two components from the Hipparcos observations (there are 249 entries related to triple and quadruple systems). On the other hand the CHARA compilation provides in several instances the observations of individual components for systems with three or more components, in which one particular pair may be associated with a Hipparcos double star solution. Because of the difficulty in making a safe and automatic identification of these components, the CHARA systems with more than two components potentially resolvable by Hipparcos were not considered in the comparison. This sample was in any case too small to affect the conclusions of this investigation.

Table 22.1. Content of the selected comparison data set between Hipparcos and the CHARA Catalogue.

Categories (see text)	1	2a	2b	3a	3b	Total
Entries in common	906	192	866	104	38	2106
With Hipparcos double star solution	765	159	637	95	24	1680
$\Delta\varrho < -50$ mas	18	3	35	3	1	60
$\Delta\varrho > +50$ mas	37	4	62	5	6	114
Systems used in the analysis	710	152	540	87	17	1506

Astrometric Parameters

To be significant the comparison between the speckle measurements (ϱ_S, θ_S) and the Hipparcos observations (ϱ_H, θ_H) must be based on nearly contemporaneous observations. As most of the systems considered in CHARA have separations less than one arcsec, and many below 0.3 arcsec, the orbital motion could be large enough to prevent a meaningful comparison at the level of one milliarcsec in the case when the Hipparcos and CHARA epochs differ by more than a few months. Several observing missions collected speckle data at the end of 1990 or in 1991, quite close to the Hipparcos Catalogue epoch of J1991.25, making the epoch difference negligible. In addition not every star in the CHARA sample exhibits a significant annual orbital motion, so that observations carried out a few months before or after the Hipparcos epoch are nonetheless useful in such a comparison. Finally when two CHARA observations bracket the Hipparcos epoch, an interpolated position could be computed at the Hipparcos epoch, provided the orbital motion was not too large over the bracketed interval.

The comparison data set was eventually separated into five categories according to the reliability of the estimation of the separation at the Hipparcos epoch:

(1) systems with at least two CHARA observations bracketing the Hipparcos epoch. Two kinds of linear interpolations were tried: the first in Cartesian coordinates $X(t) = \varrho_S \sin\theta_S$ and $Y(t) = \varrho_S \cos\theta_S$, from which the separation and position angle were then computed for the Hipparcos Catalogue epoch $T_0 = $ J1991.25; the second method interpolated directly the polar coordinates. It is obvious that for a nearly circular motion the latter is preferable while for a nearly linear motion with a large excursion in position angle the interpolation in rectangular coordinates gives better results. For objects showing a motion of few degrees in position angle the two methods are equivalent at the milliarcsec level. It was found that the statistical analysis did not depend very much on the interpolation method, although for a small number of systems the cartesian and polar interpolation may lead to quite different results. The results below refer to the interpolation in Cartesian coordinates;

(2) systems for which the last CHARA observation was made earlier than T_0, with the following two sub-cases:

  (2a) the last observation was made after $T_0 - 0.5$ yr;

  (2b) the last observation was made before $T_0 - 0.5$ yr;

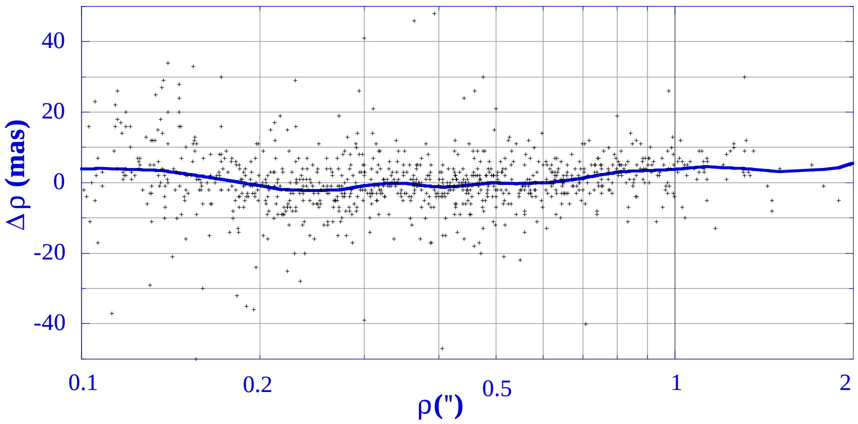

Figure 22.2. *Difference in the apparent separation of double stars observed by Hipparcos and speckle interferometry. A smooth curve has been fitted to the data points to show the systematic differences. The difference is defined in the sense Hipparcos minus speckle.*

(3) systems for which the first CHARA observation was made later than T_0, with the following two sub-cases:

  (3a) the first observation was made before $T_0 + 0.5$ yr;

  (3b) the first observation was made after $T_0 + 0.5$ yr.

In case 2 the last observation found in the CHARA Catalogue was retained for the comparisons, and in case 3 the first observation in the CHARA Catalogue was used. Attempts to extrapolate from a polynomial fit over the last (case 2) or first (case 3) two or three observations were rapidly abandoned as being too difficult to handle safely.

For all the observations the possible 180° ambiguity between the speckle observations and the Hipparcos solutions was removed by adding 180° to the position angle of the speckle data (θ_S) whenever $\cos(\theta_S - \theta_H) < -0.85$. It turns out that among the 949 'good' systems of categories 1, 2a and 3a appearing in the last line of Table 22.1, the differences in position angle left no room for ambiguity as to when a 180° shift had to be applied. The actual distribution of $\Delta\theta = \theta_S - \theta_H$ had a core of 771 systems with $-20° < \Delta\theta < 20°$ and then two distinct small populations at ±180° with respectively 95 and 83 systems. These are indeed rather small numbers considering the difficulty of removing the 180° ambiguity in the speckle observations.

The content of each category is shown in Table 22.1. In some cases the components considered in the Hipparcos solution were not the same as the two components given in CHARA. For the subsequent statistics all the systems for which the difference in the separation supplied by Hipparcos and CHARA was larger than 50 mas were removed, the difference being considered as too large to belong to the statistical distribution. This indicates an incorrect identification of the system or of the components of a multiple system, or was the consequence of an invalid interpolation of the CHARA data to the Hipparcos epoch. In Table 22.1 the occurrence of more cases of $\Delta\varrho$ in the positive wing follows from the fact that the difference was taken in the sense $\varrho_H - \varrho_S$ and that for multiple hierarchical systems CHARA usually refers to the close pair and Hipparcos to the distant component. Such systems obviously had to be removed from the comparison sample.

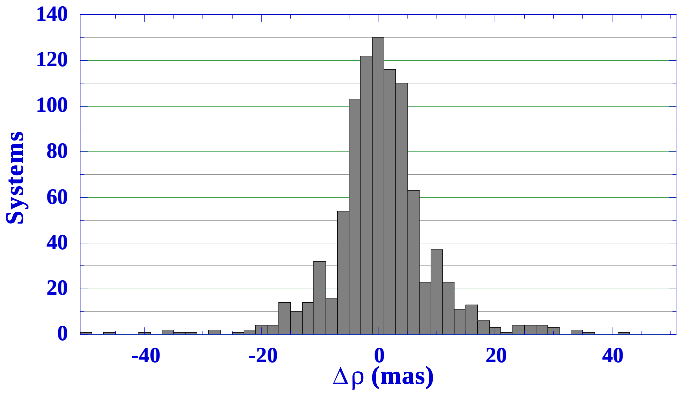

Figure 22.3. *Distribution of the differences in separation between the Hipparcos observations and the speckle measurements interpolated at the Hipparcos epoch when possible (see text). The difference is defined in the sense Hipparcos minus speckle.*

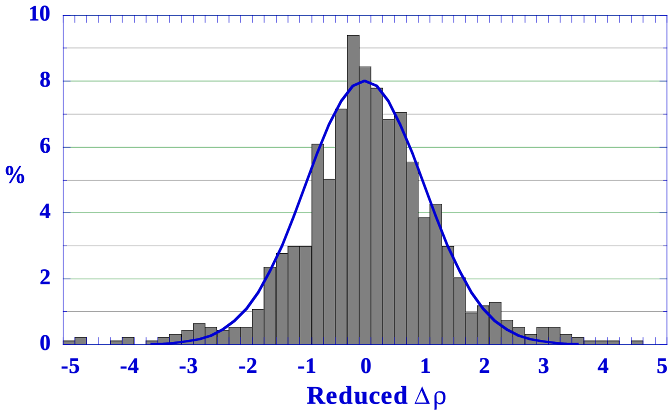

Figure 22.4. *Reduced distribution of the difference in separation between the Hipparcos observations and the speckle measurements. The solid line is the normal distribution of zero mean and unit variance. The difference is defined in the sense Hipparcos minus speckle.*

Results and Analysis of the Comparisons

For each of the above categories various analyses were carried out. Table 22.2 summarises the results for the differences $\Delta\varrho = \varrho_{\rm H} - \varrho_{\rm S}$ for each category. As expected the best results are obtained in the first group, when an interpolated separation was computed at T_0. The scatter in $\Delta\varrho$ measured by the standard deviation is 8.8 mas, close to the typical error of the Hipparcos measurement of 6–7 mas for this sample of bright stars with small magnitude difference (Figure 22.1). The slight bias is hardly significant for a population of 700 objects. The scatter is slightly larger in categories 2a and 3a, with the selected observations within six months of the Hipparcos epoch and is much larger for the other two populations. In this last two cases the filtering at $|\Delta\varrho| < 50$ mas makes the scatter somewhat too optimistic, since differences may fall in the range 50 to 100 mas.

Table 22.2. Summary statistics of the comparison of the separations between Hipparcos and speckle interferometry.

Categories (see text)	1	2a	2b	3a	3b
Number of systems considered	710	152	540	87	17
Mean of $\Delta\varrho$ in mas	+0.5	+0.8	+1.6	+1.1	−1.9
Median of $\Delta\varrho$ in mas	0	+2	+2	+1	−4
Standard deviation of $\Delta\varrho$ in mas	8.8	9.1	14.7	11.7	13.8

The plot in Figure 22.2 shows the data points as a function of $\varrho = (\varrho_H + \varrho_S)/2$ and the running median as a solid line. There are three regimes in this plot. At the smallest separations (< 0.2 arcsec, close to the Hipparcos detection limit), there is a systematic difference of 3–4 mas but also more scatter in the data than for the larger separations. For separations between 0.2 and 0.6 arcsec, there is no noticeable systematic difference. For still larger separations, there is again an increasing systematic difference, reaching a maximum of about 3–4 mas at $\varrho \simeq 1$ arcsec. If this latter effect is real, its origin in either the Hipparcos or the speckle data is still unknown, and its full understanding requires further investigation.

Another presentation of the residuals is shown in the histograms of Figure 22.3 and Figure 22.4. The first diagram represents the distribution of the differences in separation in mas, while the second histogram gives the reduced distribution, determined by computing for each star the scaled difference as:

$$\frac{\varrho_H - \varrho_S}{\sqrt{\sigma_H^2 + \sigma_S^2}} \qquad [22.3]$$

with $\sigma_S = 3$ mas. For a normal distribution of the errors with the above variances, the scaled difference should follow a normal law with zero mean and unit standard deviation, shown by the solid line in Figure 22.4. The standard deviation of the observed scaled difference is however 1.15, slightly larger than expected and primarily due to the populated tails rather than the distribution between -2 and +2. If the standard errors in the speckle observation are accepted to be less than 5 mas, including the uncertainty induced by the interpolation at T_0, this may indicate that the quoted Hipparcos errors are too small by about 15 per cent, at least to account for the wings of the distribution.

The comparison in position angle shows that there is no systematic orientation difference larger than $0\overset{\circ}{.}05$–$0\overset{\circ}{.}1$. The scatter diagram in Figure 22.5 shows $\varrho\Delta\theta$ as a function of the separation with the median smoothed out in the solid line. A systematic difference in orientation would show up as a trend in $\varrho\Delta\theta$ such that a difference of 1 mas for $\varrho = 1$ arcsec would correspond to $0\overset{\circ}{.}05$ in orientation. The other features in this diagram less than 1 mas are not significant.

The summary statistics of $\varrho\Delta\theta$ are given in Table 22.3 for each of the categories. The number of systems in each category is smaller than the corresponding numbers in Table 22.2, because an additional filtering has been applied whenever $|\varrho\Delta\theta| > 50$ mas, to be consistent with the analysis of separations. As expected the same behaviour as in Table 22.2 is observed, with the smallest scatter in the first category linked to the interpolated positions. However the standard deviations are larger than in the case of $\Delta\varrho$, which could be explained by the fact that the apparent orbits are more circular than

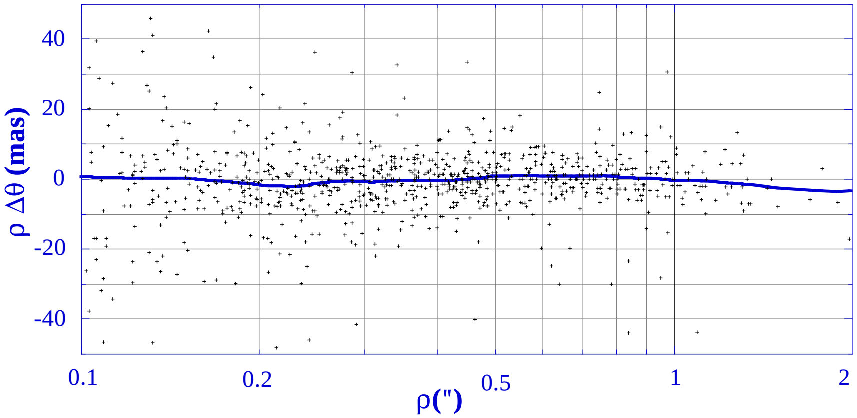

Figure 22.5. *Difference in the position angle measured by $\varrho\Delta\theta$ in mas for the double stars observed by Hipparcos and speckle interferometry. A smooth curve has been fitted to the data point to show the systematic differences. The difference is defined in the sense Hipparcos minus speckle.*

Table 22.3. Summary statistics of the comparison of the position angles between Hipparcos and speckle interferometry.

Categories (see text)	1	2a	2b	3a	3b
Number of systems considered	695	152	507	86	16
Mean of $\varrho\Delta\theta$ in mas	−1.1	+1.1	+1.7	+0.1	−0.6
Median of $\varrho\Delta\theta$ in mas	−0.5	+1.2	+2.1	+0.1	+1.8
Standard deviation of $\varrho\Delta\theta$ in mas	10.4	11.8	14.4	10.5	17.2

elongated ellipses and an error of a few months between the Hipparcos and CHARA shows up primarily in $\varrho\Delta\theta$ rather than in $\Delta\varrho$.

Figure 22.6 shows the difference of relative position of the secondary with respect to the primary computed as:

$$\Delta X = (\varrho_H \sin\theta_H - \varrho_S \sin\theta_S) \qquad [22.4]$$

$$\Delta Y = (\varrho_H \cos\theta_H - \varrho_S \cos\theta_S) \qquad [22.5]$$

There is no preferred orientation in this plot and the standard deviations in each direction are nearly identical, respectively 10.2 and 10.4 mas.

22.3. Relative Photometry

Through the history of the double star observations little attention has been given to provide magnitude differences of high quality, at least in comparison with the efforts made to get reliable astrometry. Lately CCD observations have dramatically changed the situation with the possibility of processing digitised images with good photometric calibrations. A large amount of such data has been made available by A.N. Argue *et al.* (1992) from observations carried out at La Palma in 1986–1987.

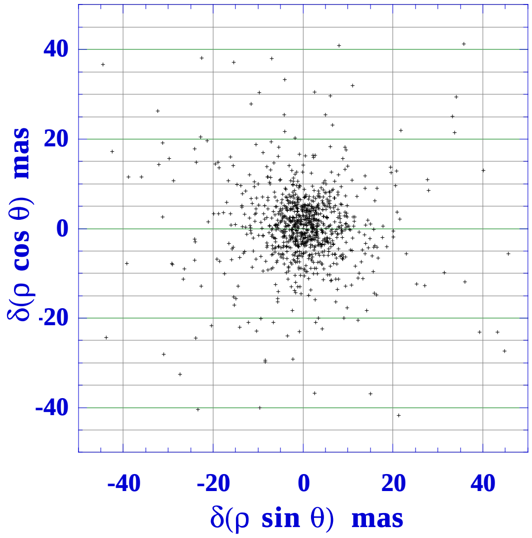

Figure 22.6. *Relative position on the tangent plane on the sky between the position of the secondary with respect to the primary obtained by Hipparcos and the speckle interferometry. The difference is defined in the sense Hipparcos minus speckle.*

More than 2300 systems were observed with separations typically larger than 0.7 arcsec and usually in the range 1 to 5 arcsec. Among them, there are 1360 systems of the Hipparcos programme. These were all detected as non-single from the Hipparcos observations and solved for the astrometry and photometry with solutions of higher quality than the average, primarily because there are no close binaries in the sample. The magnitude difference range covers the whole Hipparcos range up to $\Delta m = 4$ mag and the reported accuracy is typically between 0.01 and 0.02 mag, smaller by a factor two to four than the Hipparcos standard error for this sample as seen in Figure 22.1.

The photometric system used in the La Palma observations is different from Hipparcos and the comparison of the magnitude differences cannot be done directly. Argue and his colleagues provide component magnitudes in the V and R wavebands of the Landolt photometric system based on the Johnson UBV and Cousins' RI systems. Writing the link between the BV and Hp bands as the function:

$$Hp - V = f[(B - V)] \qquad [22.6]$$

the magnitude difference between the components A and B is given by:

$$\Delta m_{\mathrm{LP}} = \Delta m_V + f\,[(B - V)_B] - f\,[(B - V)_A] \qquad [22.7]$$

where Δm_{LP} is the La Palma magnitude difference in the Hp band. For main-sequence stars there is a correspondence between $B - V$ and $V - R$, which in principle allows the transformation of the $V - R$ of each component as measured by Argue *et al.* into $B - V$. Only systems with $|\Delta m_V - \Delta m_R| < 0.4$ mag have been selected in order to ensure that Equation 22.7 gave a good approximation to the Hipparcos system. Finally the risk of possible misidentification was limited by excluding from the analysis all the systems with differences in separation between Hipparcos and La Palma larger than 0.3 arcsec. In the end, the comparison sample reduced to 958 systems.

The main results of the comparison are plotted in the two diagrams of Figure 22.7 with $\Delta m_{\mathrm{LP}} - \Delta m_{\mathrm{H}}$ in ordinate and, in abscissa, the magnitude difference (upper panel)

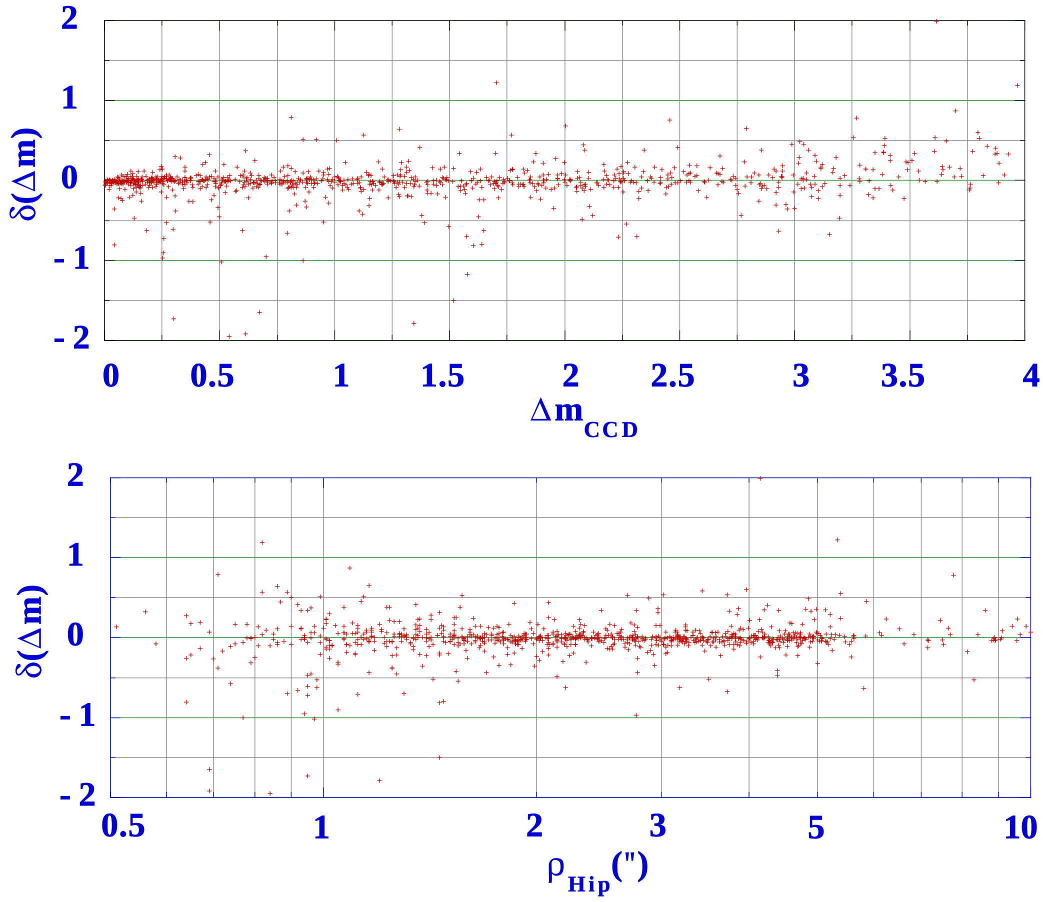

Figure 22.7. *Comparison of the magnitude difference between the components of double stars observed by Hipparcos and by ground-based CCD at La Palma, shown as a function of the magnitude difference (upper diagram) and of the separation (lower diagram).*

and separation (lower panel). The average of $\delta(\Delta m) = -0.002$ mag and the standard deviation is 0.13 mag. Up to $\Delta m \simeq 3$ mag there are no systematic differences between the Hipparcos and ground-based measurements. Neither is the separation a factor affecting the difference, at least for $\varrho > 1$ arcsec. For smaller separations the scatter is larger and is more likely due to the CCD observations which are less reliable in this range.

Regarding the distribution of the reduced differences:

$$\frac{\Delta m_{\rm LP} - \Delta m_{\rm H}}{\sqrt{\sigma^2_{\rm LP} + \sigma^2_{\rm H}}} \qquad [22.8]$$

plotted in Figure 22.8, it is quite different from a normal law of zero mean and unit variance. There is a small systematic zero effect of -0.2 mag in reduced values, or -0.01 mag in unscaled values, which is acceptable. The scatter of the distribution however is much larger than expected if the quoted standard deviations are real estimates of the random errors. A normal curve with standard deviation 1.4 provides a good fit to the central part of the observed distribution, but does not account for the tails. Clearly the reduced distribution is not Gaussian and exhibits extended wings. The effect introduced by the difference between the two photometric systems is probably

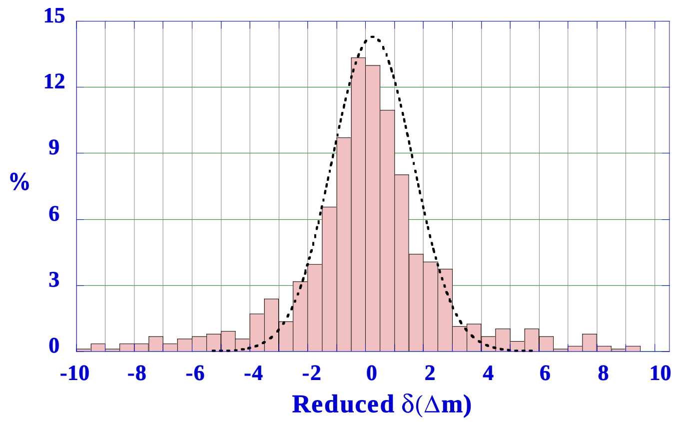

Figure 22.8. *Histogram of the reduced difference of the relative photometry between Hipparcos and the CCD observations of La Palma. The dotted line is a normal distribution of zero mean and standard deviation of 1.4, which best fits the core of the data.*

non-negligible at the level of few 0.01 mag and contributes also to increase the random scatter in a way hard to quantify. Another natural source of increased scatter is stellar variability. While the Hipparcos data are averages over a three-year period, the La Palma measures are much more liable to instantaneous deviations in magnitude.

22.4. Conclusions

The above comparisons have been restricted to the relative astrometry and photometry, for which ground-based data of comparable quality exist. The comparisons confirm the excellent overall quality of the Hipparcos results in the astrometry of double stars. The comparison of absolute astrometry was not possible at the same level, because of the lack of an independent sample matching the Hipparcos quality. However there is no real difference between the absolute astrometry of single stars and that of double and multiple stars, except that the latter are not as accurate. The confidence in the astrometry of single stars applies equally well to the double and multiple stars. In particular there are no reasons to suspect that the quoted standard errors are underestimated by more than 10 to 20 per cent.

For the photometry the situation is not so clear and illustrates the loss of accuracy in disentangling the complex signal of a multiple system into that of its components. While the photometry of the single stars (Chapters 14 and 21 of this Volume) is precise and accurate and limited primarily by the photon noise for star fainter than 8 mag, no such feat was achieved for the relative photometry of double and multiple systems. However, it was not possible to assess exactly what kind of systematic effects are to be expected and whether the overall underestimation of the standard errors applies equally to all the stars.

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