19. COMPARISONS WITH GROUND-BASED ASTROMETRY

The Hipparcos Catalogue constitutes the best materialization of the optical reference frame with a precision of the order of 1 mas in position and 1 mas/yr in proper motion. It will supersede several astrometric catalogues currently in use such as the FK5 and the PPM. In this chapter an analysis of the FK5 and PPM positions and proper motions against Hipparcos allows the global rotations existing between the Hipparcos(ICRS) and the FK5 and PPM systems to be derived. Beyond the global rotation, systematic and regional effects as large as 100 mas, functions of the right ascension and declination, are found and described. Analyses of new reductions of astrolabe and meridian observations using Hipparcos astrometry are included as well as an assessment of the astrometric measurements carried out with the Mark III interferometer.

19.1. Introduction

The Hipparcos Catalogue is referred to the International Celestial Reference System (ICRS) and constitutes the optical counterpart of the inertial system materialised by a set of radio sources observed by the VLBI technique. Its internal precision is typically below 1 mas in right ascension and declination for all the bright stars and about 1 mas/yr for the components of the proper motion. In addition it is expected to be free of regional errors at the level of 0.1–0.2 mas, a level much lower than any existing global catalogue. This provides the opportunity of using Hipparcos astrometry as a virtually error-free reference to determine the true errors of other catalogues at the Hipparcos epoch and to devise rules to correct for their systematic errors.

Up until now, the FK5 provided the basic stellar reference frame as adopted by the IAU in 1976. It was considered to be the best realisation of a stationary system through the accurate coordinates and proper motion of 1535 bright stars (Fricke *et al.* 1988). Although the ICRS is nominally consistent with the FK5 for the mean equinox and equator of the standard epoch J2000, there is still a global rotation between the two reference systems due to the uncertainty of the stellar positions in the FK5.

Beyond this rotation, regional systematic differences between the FK5 stars (and hence PPM) and the Hipparcos positions do exist up to 100 mas. In this chapter these systematic differences are evaluated and characterized as a function of the right ascension and declination. Due to the global nature of the construction of the Hipparcos Catalogue and its intrinsic accuracy (on a global scale better than 0.2 mas) there is no doubt that these regional differences must be attributed to the FK5 (and hence PPM) and not to Hipparcos. Eventually the knowledge of these differences might allow old astrometric observations tied to the FK5 frame to be reprocessed.

The PPM Catalogue constitutes the most up to date (to be superseded by the availability of the Tycho Catalogue) compilation Catalogue for the position and proper motion of reference stars. Although it was nominally constructed in the same frame as the FK5, it is desirable to determine its main systematic differences with respect to Hipparcos independently and to assess the external error in position at the epoch J1991.25 and, more important, the true quality of its proper motions.

There is an important difference between the comparison of the FK5 and the PPM to the Hipparcos results. In the former situation, all the FK5 stars are in the Hipparcos solution, so there is no risk of bias due to a truncated distribution of the Catalogue. In the case of the PPM, there is only one quarter of the PPM stars (see Section 19.3) used in the comparison (because the others are not in the Hipparcos Catalogue). Therefore the sky distribution of this sample reflects only the selection made for the Hipparcos programme and deviates from the PPM as a whole.

19.2. Comparison with the FK5 Catalogue

The FK5 Stars

The compilation of the FK5 represented a major effort by the Astronomisches Rechen-Institut to provide a realization of a dynamical frame from the analysis of more than 300 individual catalogues primarily observed with meridian circles. It represents a revision of the FK4 and results from the determination of systematic and individual corrections to the mean positions and proper motions of the FK4, the elimination of the error in the FK4 equinox, and the introduction of the IAU (1976) system of astronomical constants. Its content of 1535 bright stars (the FK5 extension to 3522 stars is not considered here because of its lower accuracy) has an expected accuracy of 0.03 arcsec at the mean epoch of the catalogue: 1955 in right ascension and 1944 in declination. The mean error quoted for the proper motion is 0.6 mas/yr for the northern hemisphere and 1.0 mas/yr for the southern. By propagating the FK5 positions directly to the Hipparcos epoch J1991.25, this leads to an expected error in the right ascension and declination of 40 to 60 mas according to the hemisphere. About 95 per cent of the stars of the FK5 have a Hipparcos magnitude in the range 2 to 7 mag, i.e. brighter than the average Hipparcos star, thus their Hipparcos accuracies are better than average.

Method of Analysis

All the 1535 stars of the FK5 have been observed successfully by Hipparcos and their positions are known at epoch J1991.25 with an accuracy typically 0.4 ± 0.1 mas in declination and 0.6 ± 0.2 mas in right ascension (Figure 19.1), the larger scatter in the latter case being the result of the strong dependence of the Hipparcos accuracy in right ascension with the declination. The corresponding figures for the proper motions are 0.7 ± 0.2 mas/yr and 0.55 ± 0.15 mas/yr with the same kind of dependence on the coordinates as the positions.

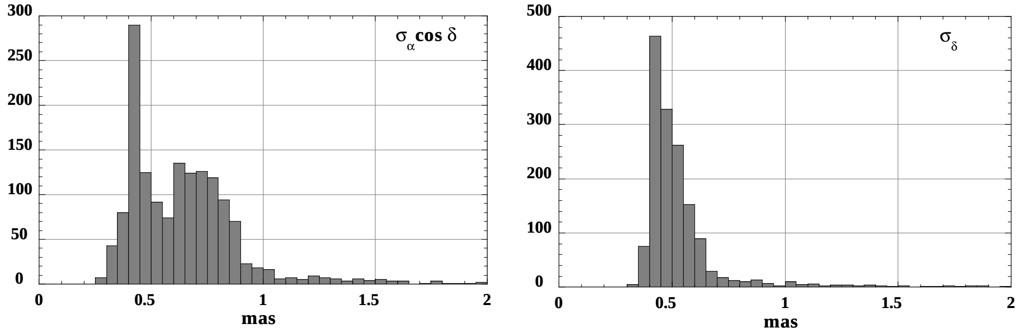

Figure 19.1. *Distribution in the Hipparcos Catalogue of the formal errors in right ascension and declination of the 1535 FK5 stars.*

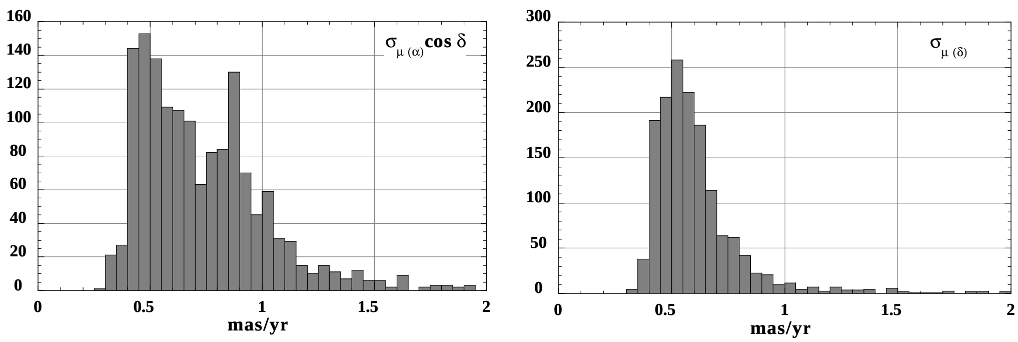

Figure 19.2. *Formal error of proper motions of the 1535 FK5 stars in the Hipparcos Catalogue.*

Data Filtering

A number of the FK5 stars have been found either to be double (97 cases) or to present a non-uniform motion (95 cases), indicating that some may actually be astrometric binaries. As a consequence the Hipparcos proper motion constructed on a timebase shorter than the orbital period might be biased. For another 78 entries the Hipparcos solution has been constructed by adding to the standard astrometric parameters one or several orbital elements as supplementary unknowns. Therefore the astrometry refers in this case to the centre of mass, which may differ from the photocentre used in the FK5; these stars were not considered reliable enough for the comparison. Finally there were 22 solutions with residuals significantly larger than the measurement error and another 10 with an apparent motion of the photocentre ascribed to the variability of one of the components of a binary star. All these stars have been excluded from the analysis, resulting in 1233 reliable solutions remaining. Most of the 302 discarded stars in fact have a good Hipparcos solution, but because of their multiplicity they may exhibit systematic differences with the FK5 positions arising from a physical origin while for the remaining 1233 single stars the differences could be accounted for as zonal errors.

Global rotation

The Hipparcos Catalogue was referred to the ICRS after the final astrometric solution had been rotated as explained in Chapter 18. Nominally the ICRS was to maintain the continuity with the previous dynamical reference system realized by the FK5 Catalogue. However due to its limited accuracy the alignment of the ICRS pole and origin of right

Table 19.1. Global orientation and spin differences between the Hipparcos and FK5 Catalogues.

Orientation (mas)	Spin (mas/yr)
$\epsilon_x = -18.8 \pm 2.3$	$\omega_x = -0.10 \pm 0.10$
$\epsilon_y = -12.3 \pm 2.3$	$\omega_y = +0.43 \pm 0.10$
$\epsilon_z = +16.8 \pm 2.3$	$\omega_z = +0.88 \pm 0.10$

ascension with the corresponding pole and origin of right ascension of the FK5 could not be achieved with consistency better than 20 mas for the pole and 80 mas for the origin of the right ascension. The final Hipparcos solution, ICRS(Hipparcos) and the optical reference frame defined by the FK5, J2000(FK5), differ by a pure rotation and numerous zonal differences of various wavelengths.

Both the rotation and zonal effects can be analysed globally by means of the decomposition of the vectors fields $[(\alpha_\mathrm{F} - \alpha_\mathrm{H}) \cos\delta,\ \delta_\mathrm{F} - \delta_\mathrm{H}]$ and $[(\mu_{\alpha*})_\mathrm{F} - (\mu_{\alpha*})_\mathrm{H},\ (\mu_\delta)_\mathrm{F} - (\mu_\delta)_\mathrm{H}]$ on a set of orthogonal vectorial harmonics. The first degree of these harmonics represents the pure rotation while the harmonics of higher degree account for the zonal differences at decreasing wavelengths with increasing degree. The global rotation and spin differences, together with their uncertainty, are given in Table 19.1. These values were used to rotate the positions and proper motions of the 1233 selected stars to the ICRS at epoch J1991.25. The remaining differences are analysed below.

Epoch transformation

The stellar positions and proper motions in the FK5 are given for the epoch J2000 in the FK5 system, while the Hipparcos Catalogue being an observation catalogue is referred to an epoch close to the average observation time, namely T_0 = J1991.25(TT). All the FK5 positions have been propagated from J2000 to the epoch T_0 using the FK5 proper motions in a straightforward manner. No attempt has been made to estimate the errors in the FK5 coordinates at the epoch T_0 for each star, using in the following discussion an overall and global estimate of the errors. In the following, these positions (rotated and propagated to T_0) are denoted by α_F, δ_F while the Hipparcos positions are labelled α_H, δ_H.

Results of the Comparison

For each of the 1233 comparison stars the positional differences:

$$\Delta\alpha \cos\delta = (\alpha_\mathrm{F} - \alpha_\mathrm{H}) \cos\delta_\mathrm{H} \qquad [19.1]$$

$$\Delta\delta = (\delta_\mathrm{F} - \delta_\mathrm{H}) \qquad [19.2]$$

have been determined and analysed from a statistical point of view. A similar approach was taken for the proper motions. Results are shown in a series of diagrams as a function of the right ascension and declination: in Figures 19.3–19.4 for the positions, and Figures 19.5–19.6 for the components of the proper motion. A fit has been made through the data points using a robust fitting technique with a moving window of 100

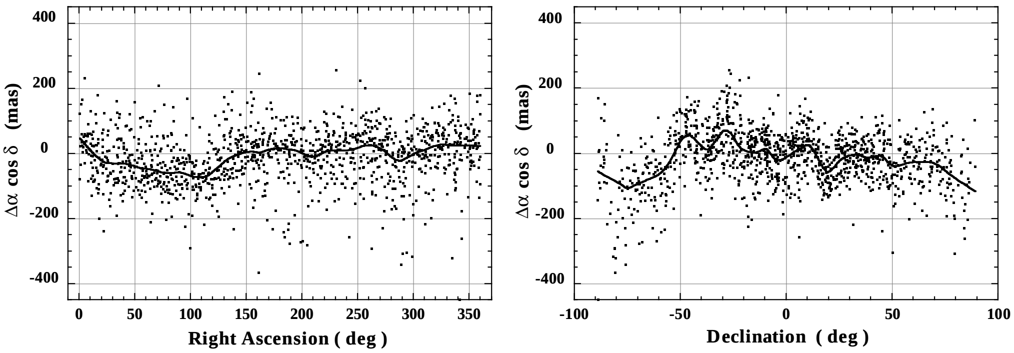

Figure 19.3. *Difference in right ascension between the FK5 and the Hipparcos Catalogue at epoch J1991.25. The solid line results from a robust smoothing of the data. Differences are in the sense FK5 – Hipparcos.*

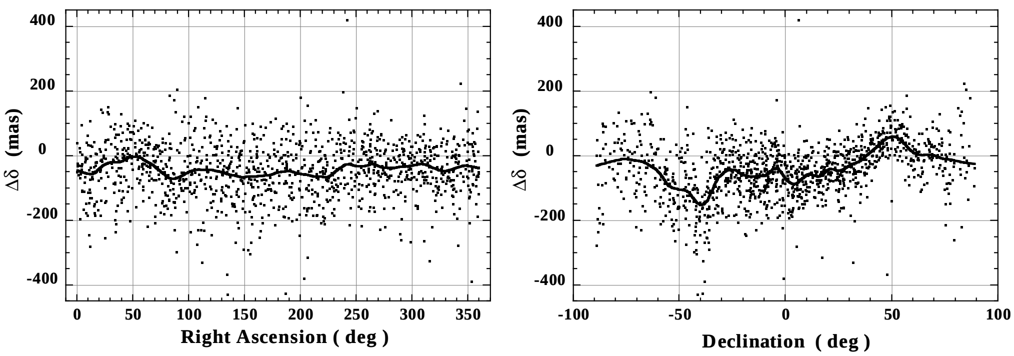

Figure 19.4. *Difference in declination between the FK5 and the Hipparcos Catalogue at epoch J1991.25. The solid line results from a robust smoothing of the data. Differences are in the sense FK5 – Hipparcos.*

data points. If the Hipparcos formal errors are correct, virtually all the scatter in the plots must originate from the FK5 positions.

There are several notable features in the plots of the positional differences between Hipparcos and the FK5:

1. the ICRS and FK5 equators are about 60 mas apart, leading to a systematic effect in declination between the two catalogues of the same magnitude. This effect is clearly seen in Figure 19.4 (in the left plot) with the average of $\Delta\delta \simeq -60$ mas;
2. both coordinates show significant regional differences as large as ±100 mas, an amplitude which is definitely larger than the expected accuracy of the FK5 at the Hipparcos epoch. Recent observations with meridian instruments have confirmed this effect and support the claim that these are local distortions in the FK5 rather than regional errors of Hipparcos;
3 both the north and south polar regions exhibit larger discrepancies and scatters than the regions at intermediate declinations;
4. the scatter in each of these diagrams is a good and robust measure of the FK5 external error at epoch J1991.25. From this analysis:

$$\sigma_{\alpha*} \sim \sigma_\delta \sim 80 \text{ to } 100 \text{ mas}$$

and:

$$\sigma_{\mu_{\alpha*}} \sim \sigma_{\mu_\delta} \sim 2.0 \text{ to } 2.5 \text{ mas/yr}$$

for the global inaccuracy, combining the random component (about 55 mas in both coordinates), and the contribution of the regional errors (which amounts to about 60 mas). The random error in declination is larger in the southern hemisphere (~ 70 mas) than in the northern (~ 50 mas). For the proper motion the random component is 1.7 mas/yr and the contribution of the zonal distortion to the standard deviation is 1.5 mas/yr, with no clear distinction with the sign of the declination.

These figures are markedly larger than the expected error at epoch J1991.25 and than the quoted uncertainty for the proper motion, even if only the random components are considered. One might have expected that locally, over a small field, the proper motion components would have been consistent below 1 mas/yr, which is definitely not the case. However the size of the fields used in this analysis are not very small (200 square degrees) as a result of the small number of stars, and the distortion on a very small scale cannot be separated from the truly random errors.

Using this uncertainty of 2 mas/yr for the proper motion the propagation from the mean observation epoch to J1991.25 yields precisely the observed uncertainty in the position as deduced from the comparison with the Hipparcos position. This consistency indicates that the standard errors found for the position and proper motion are broadly correct and that the external accuracy, including zonal errors, of the FK5 is not as good as has been believed. This discrepancy has already been pointed out by Morrison *et al.* (1990) from their meridian observations;

5. the regional errors in proper motion behave in a similar manner to the positional errors (as a function of declination). For example, the overall shapes of the curves representing $\Delta\alpha\cos\delta$ and $\Delta\mu_\alpha\cos\delta$ as a function of the declination are rather similar. The same is true for the declination and the corresponding proper motion. Since the FK5 positions are propagated from the mean observation epoch of the FK5 to J1991.25, i.e. over about 50 years, a local error of 2 to 3 mas/yr in proper motion gives rise to a local distortion in the position of about 100 to 150 mas at the same latitude. Thus, the wavy pattern in the positional differences with the declination might be simply a consequence of the zonal error in proper motion. No similar correlation can be drawn from the analysis as a function of right ascension.

Further Investigations

The complete characterisation of the regional distortions of the FK5 needs to be investigated more deeply, in particular in order to classify the various errors according to their characteristic scale over the sphere. As noted previously a decomposition on the vectorial harmonics on the sphere has been used to determine the global rotation and spin and the associated uncertainties. In addition, it appears clearly that most of the power spectrum lies in the harmonics of degree 1 and 2, both in their spheroidal and toroidal form (Mignard & Morando 1989). A complete investigation will allow easy-to-use formulae that will be sufficient to describe the main zonal errors of the FK5 frame to be derived.

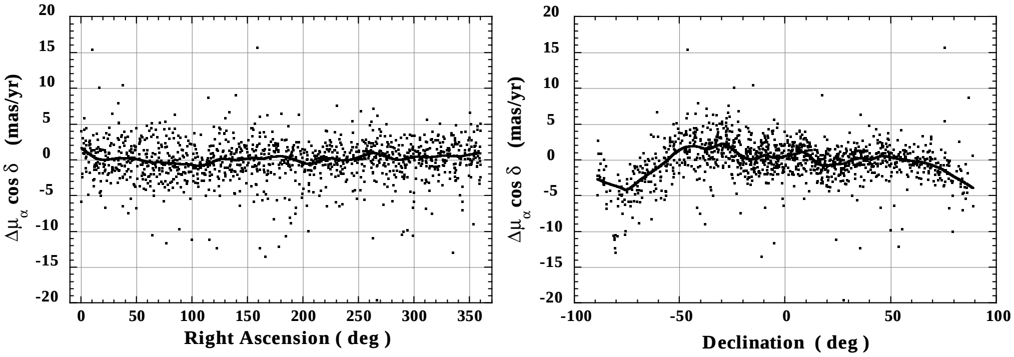

Figure 19.5. *Difference in proper motion in right ascension between the FK5 and the Hipparcos Catalogue at epoch J1991.25. The solid line results from a robust smoothing of the data. Differences are in the sense FK5 – Hipparcos.*

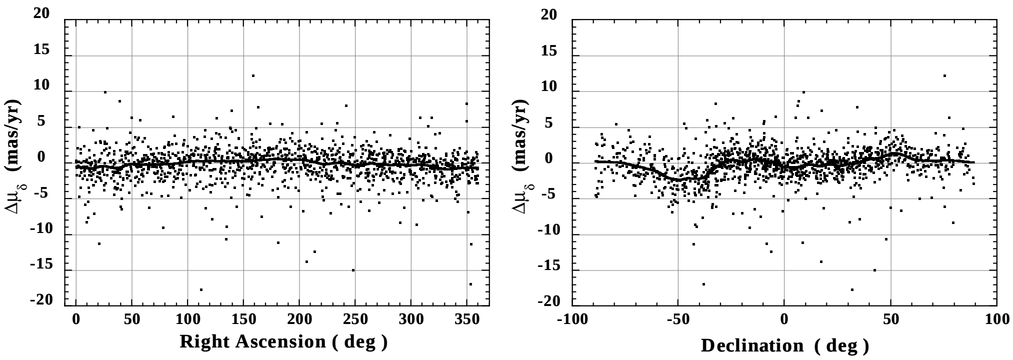

Figure 19.6. *Difference in proper motion in declination between the FK5 and the Hipparcos Catalogue at epoch J1991.25. The solid line results from a robust smoothing of the data. Differences are in the sense FK5 – Hipparcos.*

19.3. Comparison with the PPM Catalogue

The basic catalogue for position and proper motion was until the early 1990's the SAO Catalogue, published in 1966. The SAO was intended for satellite-tracking purposes and contains a compilation of positions and proper motions for 258 997 stars reduced to a common system, nominally the B1950(FK4) coordinate system. At epoch 1990 the typical errors in position and proper motion were respectively 1 arcsec and 1.5 arcsec/century. The PPM Catalogue (Röser & Bastian 1991; Bastian & Röser 1993) was designed to provide a more accurate net of reference stars on the J2000(FK5) system based on multiple positional epochs rather than the usual two in the SAO.

The PPM Stars

The PPM north gives the J2000 positions and proper motions for 181 731 stars north of a declination $-2\overset{\circ}{.}5$ and brighter than 10.5 mag, although a small sample of fainter stars is included. The published mean error of positions at epoch J1990 and proper motions are respectively 0.27 arcsec and 0.43 arcsec/century. The PPM south covers the rest of the celestial sphere and comprises 197 179 stars with an astrometric precision of 0.11 arcsec for the positions at J1990 and 0.30 arcsec/century for the proper motions up to a magnitude of 10.5 mag, with few fainter stars. Both catalogues are constructed to represent as closely as possible the reference frame defined by the FK5.

There are 108 046 Hipparcos stars in the two PPM Catalogues, 54 801 in PPM north and 53 245 in PPM south respectively. Most of the faint stars of the Hipparcos programme, $Hp > 10.5$ mag are missing in the PPM and so are not included in this analysis. Unlike the FK5, the comparison sample is now, but for the faintest stars, the same as the whole Hipparcos Catalogue but only 25 per cent of the PPM content. Thus the error distribution (on the Hipparcos side) as a function of the magnitude and of the position on the sky is the same as the Hipparcos distribution, of the order of 1 mas and 1 mas/yr for the position and proper motion, virtually error-free compared to the PPM accuracy.

The Global Rotation

As noted previously, the PPM is nominally in the FK5 system, which implies that the global rotation between the Hipparcos(ICRS) and the PPM should be given by the rotational parameters of Table 19.1. An analysis of the systematic differences:

$$\Delta\alpha \cos\delta = (\alpha_{\rm P} - \alpha_{\rm H}) \cos\delta_{\rm H} \qquad [19.3]$$

$$\Delta\delta = (\delta_{\rm P} - \delta_{\rm H}) \qquad [19.4]$$

by decomposition of a set of orthogonal vectorial harmonics gives the global rotation as the component of first degree in this representation. A similar decomposition for the differences in proper motions yields the spin components. Results are shown in Table 19.2 and are relatively consistent with the rotation of the FK5, at least for the components ϵ_x and ϵ_z, and slightly outside the probable error for ϵ_y. This confirms that, as far as positions are concerned, the PPM Catalogue is globally aligned with the FK5 system, which was not obvious to achieve owing to the very small density of FK5 stars. For the rotation rate, the differences are more significant, i.e. above the 3σ level.

Despite the much larger number of stars in the PPM comparison compared to the FK5, the global rotation is defined with exactly the same accuracy, indicating again that there is probably no single global rotation valid for all categories of stars. The technique of harmonic analysis is naturally global and does not permit a rotation to be defined separately for the north and the south. Therefore, both the values and the formal standard errors of the global rotation and spin parameters given in Table 19.2, should not be taken too literally. Since the zonal deviations reach a few tenths of an arcsec, the concept of a 'PPM system' is loosely defined at the level of a few mas. If, instead of the Hipparcos Catalogue, the Tycho Catalogue had been used for the comparison (the latter being strictly on the Hipparcos system), a different result might have been obtained due to the very different distribution of the stars on the sky. The comparison sample would have been the entire PPM with its clear-cut concentration towards the galactic plane.

The number of stars allows the global rotation to be studied according to different star groupings. As an illustration, an analysis of the rotation per interval of magnitude is shown in Figure 19.7. There is a clear trend in ϵ_z indicating a variable position of the origin in right ascension according to the star brightness. There is also a small trend in ϵ_x but this is less prominent. The standard error for each data point is of the order of

Table 19.2. Global orientation and spin differences between the Hipparcos and PPM Catalogues.

Orientation (mas)	Spin (mas/yr)
$\epsilon_X = -22.5 \pm 2.4$	$\omega_X = -0.66 \pm 0.07$
$\epsilon_Y = -\ \ 7.0 \pm 2.4$	$\omega_Y = +0.84 \pm 0.07$
$\epsilon_Z = +16.8 \pm 2.4$	$\omega_Z = +0.14 \pm 0.07$

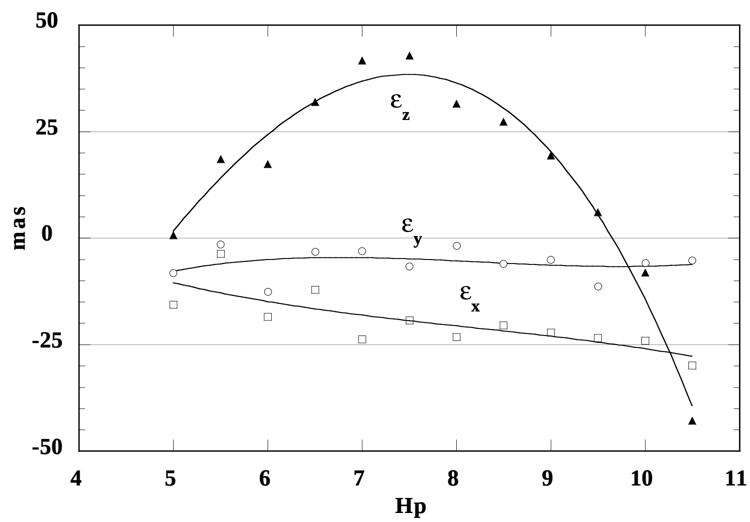

Figure 19.7. *Global rotation of the PPM with respect to Hipparcos for each class of magnitude.*

4 mas for the most populated classes in the magnitude range 7.5 to 9.5 mag, well below the level of variation of the rotation components.

Regional Differences

The differences between PPM and Hipparcos have been determined at J1991.25 by computing PPM positions at this epoch with the PPM proper motions. The global rotation and spin have been removed from the differences in Equations 19.3–19.4 and the remaining residuals have been analysed as a function of the right ascension and declination in the same way as for the FK5. Results are shown in Figures 19.8–19.10 for the positions and in Figures 19.11–19.13 for the proper motions.

The following features are noted:

1. not surprisingly the overall patterns are the same as for the differences between the FK5 and Hipparcos, in particular in the combination of the north and south catalogue. There are however noticeable differences between the two PPM Catalogues. For example, in Figure 19.8 there is a shift of about 50 mas between the PPM north and south in the systematic difference $\Delta\alpha\cos\delta$ between the PPM and Hipparcos. Otherwise the curves are rather alike. In Figure 19.9, one sees again the effect observed with the FK5 regarding the position of the FK5 equator compared to the ICRS. The departure of the order of −50 mas in the north and −80 mas for

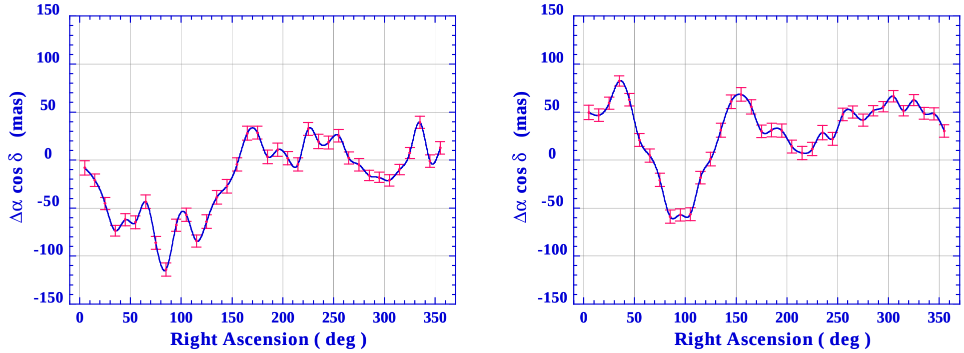

Figure 19.8. *Median difference in right ascension between the PPM and the Hipparcos Catalogue at epoch J1991.25 as a function of the right ascension. PPM north on the left and PPM south on the right.*

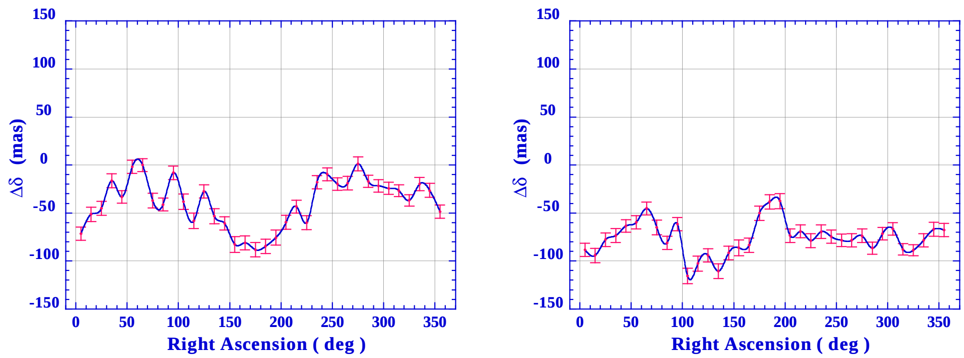

Figure 19.9. *Median difference in declination between the PPM and the Hipparcos Catalogue at epoch J1991.25 as a function of the right ascension. PPM north on the left and PPM south on the right.*

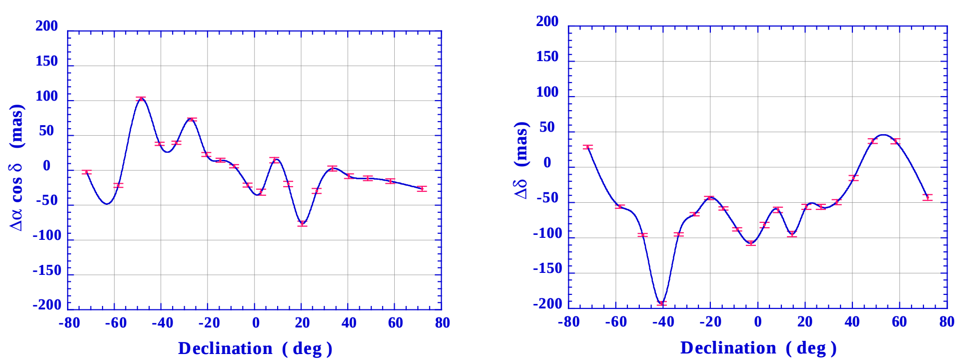

Figure 19.10. *Median difference in right ascension and declination between the PPM and the Hipparcos Catalogue at epoch J1991.25 as a function of the declination. Difference in right ascension on the left and in declination on the right.*

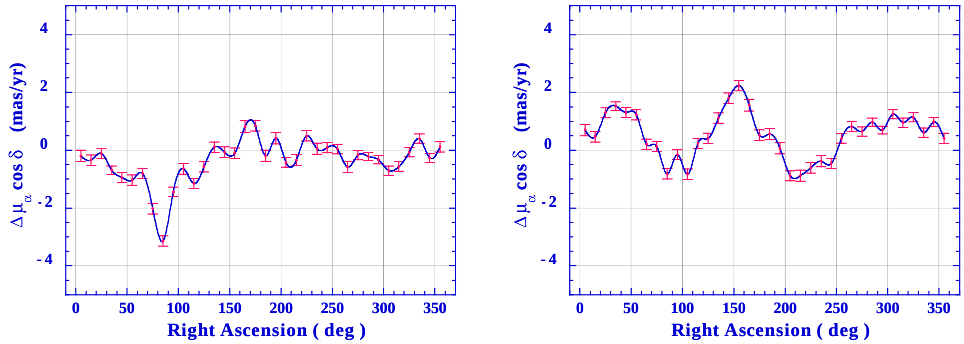

Figure 19.11. *Median difference in proper motion in right ascension between the PPM and the Hipparcos Catalogue at epoch J1991.25 as a function of the right ascension. PPM north on the left and PPM south on the right.*

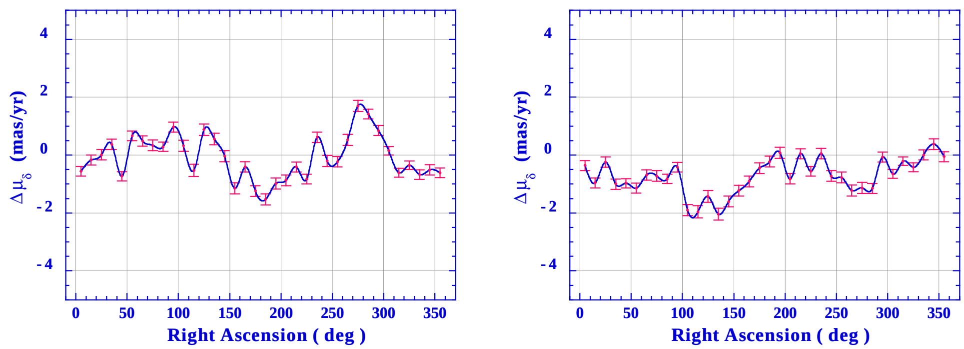

Figure 19.12. *Median difference in proper motion in declination between the PPM and the Hipparcos Catalogue at epoch J1991.25 as a function of the right ascension. PPM north on the left and PPM south on the right.*

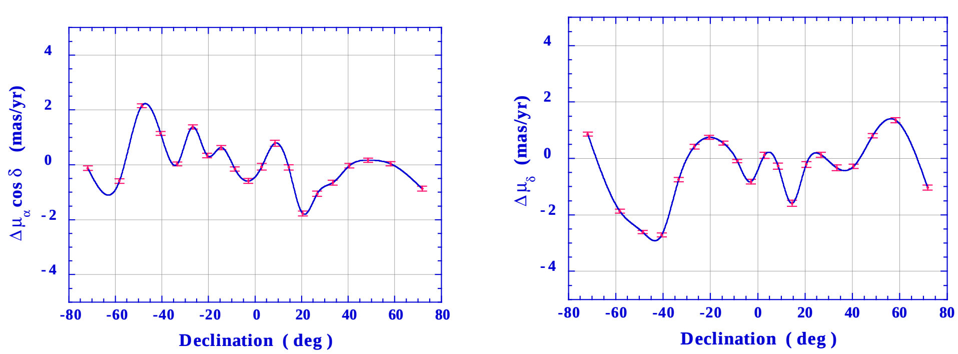

Figure 19.13. *Median difference in proper motion in right ascension and declination between the PPM and the Hipparcos Catalogue at epoch J1991.25 as a function of the declination. Difference in right ascension on the left and in declination on the right.*

Table 19.3. External accuracy (1σ) of the PPM for the random, regional and global components. The figures apply to both right ascension ($\sigma_\alpha \cos\delta$) and declination (σ_δ), and similarly for the proper motion components.

	Positions (mas)		Proper Motions (mas/yr)	
	North	South	North	South
Random	215	125	4.5	3.7
Regional	85	105	2.8	2.6
Global	230	165	5.3	4.5

the south. The difference as a function of the declination is comparable to that observed in the difference FK5–Hipparcos;

2. in the analysis as a function of declination, the error bars in positions are significantly smaller in the south than in the north (this is not obvious with the resolution of the plots, but these are typically 2.5 mas for the south and 3.5 mas for the north for the error of the median for each class of about 5000 stars). The effect is less pronounced, although visible, for the proper motion (respectively 0.07 mas/yr and 0.08 mas/yr). This is in agreement with the *a priori* better quality of the PPM south compared to the north;

3. on a more quantitative basis the random scatter about the median can be used to assess the true precision of the PPM Catalogues. The standard error on the median multiplied by the square root of the population of the class provides a robust estimate of the scatter in the core of the distribution, excluding automatically the outliers. From the analysis of both the right ascension and declination as a function of the right ascension one gets 235 mas for the north and 145 mas for the south. For the analysis as a function of declination the figures are respectively 220 mas and 155 mas. For the north this is slightly better than the quoted precision of 270 mas, but for the south the above figures are somewhat larger than the 110 mas given in the presentation of the PPM. The results of an attempt to discriminate between random and regional errors are given in Table 19.3 for the difference in position and proper motion between the PPM and Hipparcos;

4. for the proper motion, the figures lead to an external accuracy of 5 mas/yr and 4.5 mas/yr for the north and south instead of 4.0 and 3.0 mas/yr for the standard error given in the presentation of the PPM. These numbers are about ten times larger than the spin rate corrections and the uncertainty on this rotation does not affect the above conclusion. The propagation over 50 years with these figures yields too large an error in the position at epoch J1991.25 in comparison with the departure of the PPM positions with respect to Hipparcos at this epoch.

   At first sight, this discrepancy appears somewhat surprising, since various uses of the PPM have indicated that the precision of the relative proper motions over small fields (such as star clusters, the Magellanic Clouds and the Cygnus superbubble region) agrees much more closely with the stated errors. Thus the discrepancy must be due to large regional errors on the scale of 5° to 10°, which could not be taken into account in the present investigation. That regional errors of the required size could indeed be present is indicated by detailed comparison of the PPM system of positions with a preliminary Hipparcos Catalogue by Lindegren *et al.* 1995 (more specifically Figure 7 of that paper).

A study of the dispersion of the differences of the proper motions PPM–Hipparcos in cells of about 10 square degrees, yields a dispersion of 4.5 mas/yr for the north and 3.7 mas/yr for the south. This scatter is a good indicator of the level of the random errors, provided that there is no large systematic effect of typical size less than a few degrees. On the other hand the inter-cell scatter represents the contribution of the systematic differences over wavelengths larger than few degrees, and amounts to 2.7 mas/yr, which eventually leads to the 4.5 to 5.3 mas/yr for the external accuracy of the PPM;

5. the medians of the systematic differences in position at epoch J1991.25 and proper motion are very significant, often larger than five times the standard errors. As in the case of the FK5, the most conspicuous feature is the systematic shift between the ICRS and FK5 equator of about 50 mas to 80 mas. There are large zonal errors (~ 100 mas) in both coordinates;
6. the relationship between the differences in proper motions and in positions is more striking than in the case of the FK5. A comparison between Figures 19.8–19.10 and Figures 19.11–19.13 is instructive in this respect. The corresponding plots in each set are very similar in shape with roughly a scale factor between them, linked to the time span between the mean epoch of the PPM (1931) and J1991.25. Unlike the FK5 this remark applies fully to both the analysis as a function of right ascension and declination. Thus the zonal errors seen in position very likely are the consequence of the unsatisfactory knowledge of the proper motions, a situation which will not be improved until the re-reduction of the Astrographic Catalogue in the Hipparcos system and its combination with the Tycho data.

19.4. Comparison with the Mark III Interferometer Results

The Mark III interferometer was set up by Shao *et al.* (1990) at Mount Wilson Observatory in the late 1970s, and became operational for astrometric observations in September 1986. Observations were discontinued in 1993 with the development of a new instrument located at Mount Palomar Observatory. It was designed for observations in amplitude and phase mode, the latter mode allowing global astrometry to be conducted by careful laser monitoring of the delay lines. The astrometric measurement carried out in 1988 over 12 FK5 stars yielded an average 1 σ error for fifty observations of the order of 10 mas in right ascension and 6 mas in declination (Shao *et al.* 1990). However the conclusion of this first run was that an extended series of measurements was needed to ascertain the true accuracy that the instrument could achieve in absolute astrometry.

Repeated measurements over four years have been obtained by Hummel *et al.* (1994) for 11 stars at four different epochs between 1988.6 and 1992.7. Measurements at two different wavelengths were used to correct for the refractive index fluctuations in the atmosphere. Although the number of stars was too small to allow an absolute determination of the declination, the analysis of the offsets with respect to FK5 led to an accuracy of 13 mas in declination and 23 mas in right ascension. Using Hipparcos data, which for these bright stars are better than 1 mas and 1 mas/yr in position and proper motions, one can see whether these estimates are realistic or not, and assess the real potential of the interferometers for the measurements of stellar positions.

Table 19.4. List of stars with astrometric solution obtained with the Mark III optical interferometer.

HIP	FK5	Name	*Hp*	α (deg)	δ (deg)
3031	19	ϵ And	4.3	9.63	+29.31
4436	33	μ And	3.9	14.18	+38.49
7607	52	–	3.8	24.49	+48.62
8903	66	β Ari	2.7	28.65	+20.80
10670	79	γ Tri	4.0	34.32	+33.84
101076	1534	–	4.1	307.34	+30.37
106481	1568	ρ Cyg	4.1	323.49	+45.59
109410	835	π^2 Peg	4.4	332.49	+33.17
111169	848	α Lac	3.8	337.82	+50.28
112748	862	μ Peg	3.7	342.50	+24.60
116805	1619	κ And	4.1	355.10	+44.33

Because of a difference in the reference system, which is difficult to resolve with a small unevenly distributed sample of stars, the comparison is done directly on the arc-lengths between stars of the Mark III programme and the corresponding arcs computed from the Hipparcos astrometric solution. This method has the advantage of being insensitive to a global rotation but also has the drawback of the lack of independence of the set of arc-lengths, which precludes a rigorous statistical analysis. In addition it is impossible to scale separately the accuracy in declination and right ascension.

The list of the stars with an interferometric astrometric solution is given in Table 19.4, which contains also other relevant parameters. Among the 11 stars, HIP 8903 = FK5 66= β Arietis was solved by Hipparcos with a significant orbital motion, and was eventually excluded from the comparison. Each of the ten remaining stars was observed at four different epochs by Mark III. For each of these observations the Hipparcos positions were computed using the Hipparcos astrometric parameters. The number of arcs was then $4 \times 10 \times 9/2 = 180$, for only $4 \times 10 \times 2 = 80$ independent measurements.

The distribution of the differences is shown in Figure 19.14 for the 180 arcs. They are ordered as follows: the arcs of HIP 3031 with HIP 4436 for the four epochs, then the four arcs of HIP 3031 with HIP 7607, and so on until the set of arcs is exhausted with the four arcs of HIP 112748 with HIP 116805.

The median of the differences is -1.2 mas, and the scatter measured by the standard deviation 32 mas. The latter is in agreement with the results on nine stars by Lindegren *et al.* (1995) in a comparison made against Hipparcos preliminary results using, for each star, a normal positions derived from the four individual observations. The median is not significantly different from zero over this small sample. The significance of the dispersion must be evaluated with reference to the uncertainty of the positions of each pair of stars, given that the Hipparcos error can be neglected.

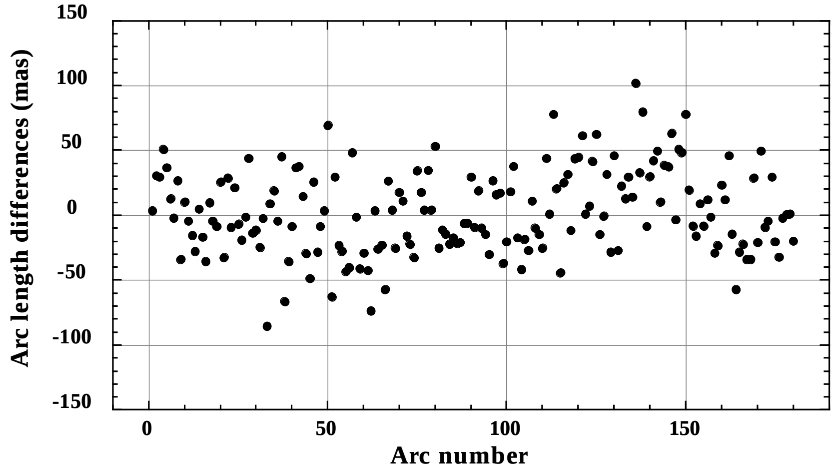

Figure 19.14. *Distribution of the differences of the arc-lengths between the Mark III observations and the corresponding arcs from the Hipparcos Catalogue.*

The propagation of the errors on the positions at each epoch to the arc-lengths was estimated by a Monte-Carlo procedure, using the mean coordinates of the ten stars to determine the reference lengths of the 45 basic arcs and then adding a Gaussian noise component on each coordinate with a standard error of 13 mas in declination and 23 mas in right ascension (σ_α and not $\sigma_\alpha \cos\delta$). From 500 such experiments it appears that the distribution of the standard error (1σ) on the arc-lengths was between 19 mas and 30 mas with a well defined average of 24 mas, somewhat smaller than the quadratic propagation of the error on the positions of the two end-points of an arc. This can be understood as follows : for two stars more or less on the same celestial meridian, the arc-length is insensitive to an error in right ascension which displaces the two stars in a direction perpendicular to the arc. A similar reasoning holds for the error in declination for two stars with similar declinations.

Thus the observed scatter of 32 mas in the arc-lengths is significantly larger than the expected value (i.e. $\simeq$ 24 mas) determined with the reported accuracy of $\sigma_\alpha = 23$ mas and $\sigma_\delta = 13$ mas, indicating that the astrometric accuracy is overestimated by about 30 per cent. One cannot conclude with certainty as to whether the right ascension or the declination should be incriminated; probably both.

19.5. Astrometric Reductions of Schmidt Plates

A preliminary study was made to test the ultimate accuracy attainable in reducing plates, using intermediate Hipparcos and Tycho 30-month solutions (Robichon *et al.* 1995). To minimise the effect of the errors on proper motions, especially for Tycho data, plates with epochs close to the Hipparcos mid-mission epoch were chosen for this test. This study has not yet been repeated using final Hipparcos and Tycho Catalogues but its results are nonetheless meaningful since the difference between the 30-month and the final solutions are much smaller than the typical error in the plate reduction.

The Observational Material: Hipparcos and Tycho 30-month Solution

The intermediate Hipparcos astrometric catalogue constructed from the first 30 months of data obtained from the satellite is described in Kovalevsky *et al.* (1995), Lindegren *et al.* (1995) and in Chapter 16 of this volume. The intermediate Tycho Catalogue version used below is T30d, the first iteration containing both proper motions and parallaxes. The properties of this version are described in Høg *et al.* (1995), Høg (1995), and in Chapters 11 and 16 of Volume 4.

The Observational Material: Schmidt Plates

The plates used in this study were taken with the CERGA and ESO Schmidt telescopes, as part of a programme on eight open clusters observed by Hipparcos. Plates were obtained in U, B, V, R, I, with a long (1–2 hours) and a short (15–30 min) exposure for each colour.

Six short-exposure plates have been used to test the ultimate accuracy attainable on positional determination from Schmidt plate measurements, and two long-exposure for comparison. Astrometric reductions were made using successively PPM data (PPM North: Röser & Bastian 1993; PPM South: Bastian & Röser 1993), and preliminary Hipparcos and Tycho data. Each field, of about 25 square degrees, contains approximately 350 PPM stars (ranging from 250 to 400), 85 Hipparcos stars (from 70 to 200), and 1000 Tycho stars (from 550 to 3000).

Mean errors of PPM positions and proper motions are of the order of 300 mas and 4 mas/yr respectively in the north (i.e. fields 1, 2, 3, 8) and 150 mas and 3 mas/yr respectively in the south (i.e. fields 4 to 7), to be compared with 1.5 mas and 1.5 mas/yr for preliminary Hipparcos results, and with 30 mas and 30 mas/yr for preliminary Tycho results. The plates were scanned with the Machine Automatique à Mesurer pour l'Astronomie (MAMA) of the Centre d'Analyse des Images in Paris.

The Results

The results obtained show that the ultimate accuracy expected from a single Schmidt plate is better than 0.10 arcsec (0.06–0.07 arcsec) for stars brighter than 11 mag, to be compared with 0.15–0.37 arcsec obtained using the PPM Catalogue. The model is more reliable when the reduction is carried out using the numerous, though less accurate data from Tycho than the more accurate, but less numerous data from Hipparcos. A decrease of the errors on the positions of the reference stars below 0.01 arcsec does not seem to improve the results. The modelling errors are of the order of 0.05 arcsec. An rms error of about 0.04 arcsec for the centring of the images is consistent with our results. This study emphasizes the importance of obtaining good proper motions for Tycho Catalogue stars, using Tycho data in conjunction with first epoch astrometric data such as the Astrographic Catalogue, in order to take advantage of these accurate data for plates taken at epochs very different from the Hipparcos mid-epoch.

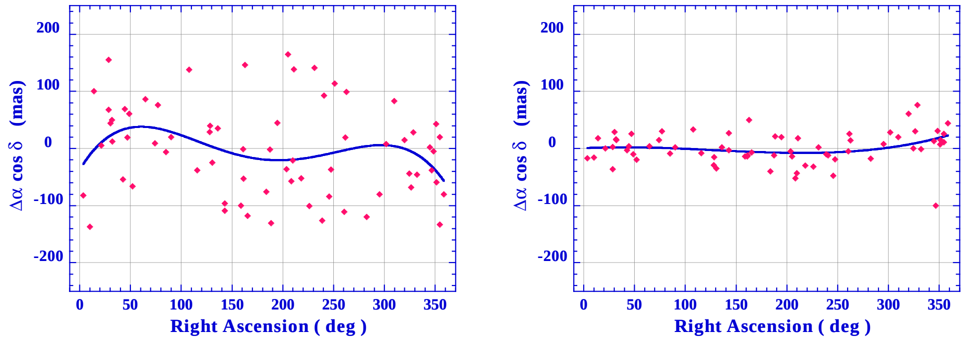

Figure 19.15. *Residual in the astrolabe catalogue after five years of observations for the right ascension of 68 FK5 stars. On the left, all the reduction has been done with the FK5 Catalogue coordinates and on the right with Hipparcos.*

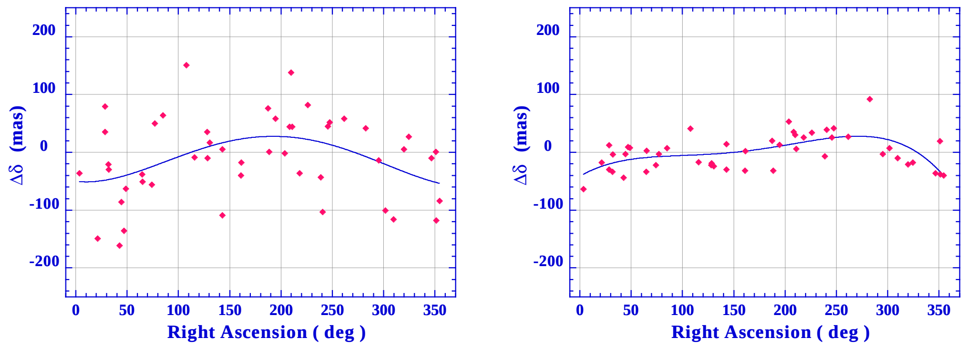

Figure 19.16. *Residual in the astrolabe catalogue after five years of observations for the declination of 48 FK5 stars. On the left, all the reduction has been done with the FK5 Catalogue coordinates and on the right with Hipparcos.*

19.6. Analysis of Recent Meridian Circle Observations

The impact of the Hipparcos and Tycho Catalogues on the meridian circle observations has been known for years. A critical examination of the residuals obtained with respect to the FK5 has been made by Réquième *et al.* (1993) with the very first positions based on the Hipparcos observations, without improvement of the proper motions. Subsequently an analysis of recent meridian circle observations, using successively the FK5 and Hipparcos as reference has been done by Réquième *et al.* (1995) with the Hipparcos 18-month solution. A remarkable improvement in the post-fit residuals of the least-squares adjustment was obtained for observations made by the Bordeaux and La Palma automatic circles. The difference between the two fits confirmed the existence of systematic errors in the FK5 and revealed small instrumental errors of 30 mas in right ascension and 50 mas in declination that were hidden by the larger noise brought about by the FK5 regional errors.

19.7. Analysis of Recent Astrolabe Observations

Astrolabe observations have been performed at OCA/CERGA on a regular basis for the last twenty years. During the last decade they were done on an impersonal photoelectric and highly automated instrument. In addition to determining the daily orientation of the local vertical, the yearly analysis of the residuals permits corrections to the star catalogue to be derived.

The processing of the same observations performed over five years centred on the Hipparcos epoch, from J1988.5 to J1993.5, has been done by using successively the FK5 coordinates and proper motions and the Hipparcos data for the same stars. The results expressed as corrections to apply to the star positions given in the astrolabe catalogue are shown in Figure 19.15 and Figure 19.16 respectively for the right ascension and declination.

The comparison between the left and right plots in each figure confirms the real improvement of the Hipparcos reference frame compared to FK5 and gives for the first time the true level of accuracy of the observations carried out with a photoelectric astrolabe. The remaining scatter in the right plots is a combination of the instrument limitation and the photon noise and should not be interpreted as errors in the Hipparcos positions.

Prior to this comparison it was difficult to decide from the residuals on the FK5 stars on the respective contribution of the instrument and that due to the FK5 uncertainties. The difference between the internal error obtained during the processing, of the order of 12 mas in right ascension and 14 mas in declination (Vigouroux *et al.* 1995), and the true external error which is in fact closer respectively to 20 mas and 28 mas is evident.

F. Mignard, M. Frœschlé, C. Turon