

Section 1.2
Astrometric Data

1.2. Astrometric Data

This section provides a framework for the interpretation of the astrometric data given in the Hipparcos and Tycho Catalogues. It explains the coordinate system, time scales and units used in the Catalogues, and in particular their relations to the standard astronomical reference systems. The precise definition of the astrometric parameters requires the introduction of several auxiliary concepts which are covered in the last two subsections.

Additional information relevant for the practical use of the astrometric data is found in Section 1.5. This includes the transformations to ecliptic and galactic coordinates and to arbitrary epochs; the calculation of space coordinates and velocities; and the propagation of errors in these transformations.

1.2.1. The Astrometric Parameters Determined by Hipparcos

The basic astrometric data in the Hipparcos and Tycho Catalogues include directions (positions), specified by celestial coordinates, their rate of change (proper motions), and trigonometric parallaxes. This is the first observational catalogue to contain parallaxes in addition to positions and proper motions for all stars in the observing programme.

The positions and proper motions are given within the International Celestial Reference System (ICRS), which is consistent with the conventional equatorial system for the mean equinox and equator of J2000, previously realised by the FK5 Catalogue (see Section 1.2.2). The timing of events is consistently based on the terrestrial time-scale, TT (see Section 1.2.3).

The standard astrometric model for a single star (Section 1.2.8) assumes uniform rectilinear space motion relative to the solar system barycentre. At some reference epoch, T_0 , the motion is then described by the following six parameters:

- barycentric coordinate direction; this is specified as right ascension, α , and declination, δ ;
- annual parallax, π , from which the coordinate distance is $(\sin \pi)^{-1}$ astronomical units;
- rate of change of the barycentric coordinate direction expressed as proper motion components $\mu_{\alpha*} = \mu_{\alpha} \cos \delta$ and μ_{δ} , in angular measure per unit time;
- radial (i.e. line of sight) velocity, V_R , in linear measure per unit time.

The radial velocity is normally obtained from spectroscopic observations. The remaining parameters, referred to as the ‘five astrometric parameters’, are determined from analysis of the Hipparcos observations and are given for almost all stars in the catalogue.

The six parameters listed above refer, in principle, to the reference epoch T_0 . The propagation of the parameters and their covariances to arbitrary epochs is treated in Section 1.5. Normally only the changes in α and δ , due to the proper motion, are appreciable. In rigorous treatment the time-dependence of all six parameters must however be explicitly considered (see Section 1.5.5). In such cases their values at the catalogue epoch T_0 are denoted α_0 , δ_0 , π_0 , μ_{α^*0} , $\mu_{\delta 0}$ and V_{R0} .

The system of units used for these data is as follows (see Sections 1.2.3 and 1.2.5):

- for α and δ when expressed as full angles: degrees (deg);
- for differential coordinates such as $\Delta\alpha^* = \Delta\alpha \cos \delta$ and $\Delta\delta$: milliarcsec (mas);
- for π : milliarcsec (mas);
- for μ_{α^*} and μ_{δ} : milliarcsec per Julian year (mas yr^{-1});
- for V_R : km s^{-1} .

Stellar distances are measured in parsec. To sufficient approximation, the distance in parsec is given by $1000/\pi$, where π is expressed in mas.

The common reference epoch used throughout the catalogue is $T_0 = \text{J1991.25(TT)}$ (see Section 1.2.6).

The standard astrometric model, using the five astrometric parameters, was found to be adequate for the majority of the stars, including the components of most resolved double and multiple stars. For several thousand apparently single stars, however, the standard model did not give an acceptable fit to the observations. These are probably binary stars, in which the centre of light describes a curved path on the sky due to the orbital motions of the components around their common centre of mass. For such ‘astrometric binaries’ additional parameters had to be introduced to describe the motion in accordance with the observations. In most cases a good empirical fit was obtained by including one or two additional terms in the Taylor expansion of the variation of the barycentric coordinate direction with time. In a few hundred cases even this was not sufficient, and Keplerian elements (sometimes partially based on existing ground-based data) were introduced to describe the orbital motion of the photocentre. These extended solutions are also given in the Double and Multiple Systems Annex.

When comparing the Hipparcos and Tycho Catalogues with previous astrometric catalogues, the following points are noted:

- positions and proper motions are given for the catalogue epoch, $T_0 = \text{J1991.25(TT)}$, which is close to the central epoch of observations (the distinction between the epoch which specifies the equinox and equator of a coordinate system, typically B1950 or J2000, and the epoch at which the star passed through the given position, should be noted);
- the International Celestial Reference System (ICRS), as represented by the Hipparcos and Tycho Catalogues, replaces the FK5 system as the practical definition of celestial coordinates in the optical region. The construction of the ICRS (see Section 1.2.2) ensures that no discontinuity larger than the uncertainty of the FK5 system occurs in the transition from FK5 (mean equinox and equator J2000) to ICRS. Thus, from the viewpoint of optical astrometry, the Hipparcos and Tycho Catalogues can be regarded as an extension and improvement of the J2000(FK5) system, retaining approximately the global orientation of that system but without its regional errors. However, it should be noted that the Hipparcos and Tycho Catalogues have *not* been tied to the FK5 system, but to the ICRS through the extragalactic radio frame (see Volume 3, Chapter 18). The relation between the ICRS(Hipparcos) reference frame and the J2000(FK5) system is further discussed in Section 1.5.7;

- providing positions within the ICRS means that they are consistent with present knowledge of the (extragalactic) radio reference system. Providing proper motions within the ICRS means that they are consistent with the best present realisations of a quasi-inertial reference system (i.e. non-rotating with respect to distant galaxies);
- since parallax is explicitly derived in the data reduction, the directions represented by the right ascension and declination at epoch T_0 are strictly barycentric. In previous astrometric catalogues, the unknown displacement due to parallax for each observation perturbed the final catalogue positions;
- at the accuracy achieved by Hipparcos, general relativity effects cannot be ignored. In the relativistic framework, the observed directions are *proper directions* referred to the Lorentzian frame moving with the observer. In order to relate the observed directions at different epochs and of different objects to a simple geometrical model of the stars and their motions, the ‘local’ effects of the moving observer (stellar aberration) and space curvature (gravitational light deflection by the Sun) are removed by transforming the proper directions to *coordinate directions* corresponding to the coordinate differences between the object and observer, expressed in the (solar system) barycentric coordinate system;
- stellar aberration has been computed from the barycentric motion of the satellite, using the full relativistic formula and combining the geocentric satellite velocities accurate to about 0.2 m s^{-1} , determined by the ground stations, with the appropriate ephemeris (see Section 1.2.4) for the barycentric motion of the Earth. Gravitational light deflection has been computed in the heliocentric metric, assuming spherical symmetry. The treatment of aberration is consistent with the IAU (1984) recommendation that the total barycentric velocity of the Earth should be used; prior to that date the elliptic component of velocity, which changes very slowly with time, was conventionally ignored;
- the proper motions are instantaneous rates of change of the barycentric coordinates at the epoch T_0 . For stars with large parallax, large proper motion, and large radial velocity, the apparent acceleration of coordinates due to perspective effects may be significant. These can be calculated on the assumption of uniform space motion relative to the barycentre, and have been accounted for, in the case of the few most significantly affected stars, in reducing the individual observations to T_0 , as described in Section 1.2.8;
- still assuming uniform barycentric motion, star positions and proper motions, within the fixed quasi-inertial reference frame of ICRS, can be calculated for arbitrary epochs, using the catalogue data and, where appropriate, taking into account the perspective accelerations. Procedures for this, and the corresponding propagation of uncertainties, are discussed in Section 1.5;
- while ground-based proper motions are typically obtained by comparing positional catalogues having an epoch difference of 20 to 50 years, the Hipparcos proper motions are determined using a temporal baseline of only a few years. The ground-based values thus give the mean motion over several decades, while the Hipparcos proper motions are, by comparison, quasi-instantaneous. This is of no consequence as long as the motions are uniform, which is a valid assumption at least for the single stars. However, many apparently single stars are in reality double, and the actual motions of the individual components or of the photocentre of the double star may be significantly non-uniform. For such objects there may be a discrepancy between the space and ground-based proper motions which is not attributable to random errors or differences of the reference frames. More extreme cases of non-uniform motion become noticeable already during the few years of the Hipparcos mission and require the extended solutions given in the Double and Multiple Systems Annex.

1.2.2. The Hipparcos Reference Frame

The positions given in the Hipparcos and Tycho Catalogues effectively define a grid of celestial coordinates (α , δ) for the epoch J1991.25. By means of the proper motions also given in the catalogues, this grid can be extended forward and backward in time to arbitrary epochs (albeit with an uncertainty that increases with the epoch difference). The positions and proper motions thus define a specific coordinate system on the sky, accessible in optical wavelengths at any epoch: the Hipparcos Reference Frame.

The Hipparcos and Tycho Catalogues have been constructed in such a way that the Hipparcos Reference Frame coincides, to within limits set by observational uncertainties, with the International Celestial Reference System (ICRS). The ICRS is practically defined by the adopted positions of several hundred extragalactic radio sources.

Construction of the ICRS: In 1995, the IAU Working Group on Reference Frames identified the celestial reference system of the International Earth Rotation Service (IERS) as the next official IAU celestial reference system under the name of ‘International Celestial Reference System’ (ICRS). The definition of the ICRS complies with the 1991 IAU recommendations on reference frames. The axes of the system are realised with an accuracy of $\pm 30 \mu\text{as}$. Their directions are consistent with those of the FK5 system (mean equinox and equator J2000) within the uncertainty of the latter ($\pm 50\text{--}80 \text{ mas}$; E.F. Arias, P. Charlot, M. Feissel & J.-F. Lestrade, 1995, *Astronomy & Astrophysics*, 303, 604), and their tie to the best current realisation of the FK5 dynamical reference system is within $\pm 3 \text{ mas}$ (W.M. Folkner, P. Charlot, M.H. Finger, J.G. Williams, O.J. Sovers, X.X. Newhall & E.M. Standish, Jr., 1994, *Astronomy & Astrophysics*, 287, 279).

In October 1995, the IAU Working Group on Reference Frames released the ICRS coordinates of some 250 extragalactic objects (out of some 600 that are monitored), constituting the International Celestial Reference Frame (ICRF). The accuracy of the coordinates of more than half of the sources is better than $\pm 0.47 \text{ mas}$. In the future, the IERS will continue to monitor the sources, issuing appropriate updates, while keeping the reference coordinates unchanged.

In order to align the Hipparcos Reference Frame with the ICRS it is necessary to find objects whose positions and/or proper motions can be referred to both systems. Unfortunately, none of the extragalactic objects defining the ICRS is bright enough at optical wavelengths for direct observation by Hipparcos. Therefore, the linking of the two systems was made *a posteriori* through auxiliary ground-based observations, as described in detail in Volume 3, Chapter 18. The estimated uncertainty of the link corresponds to a standard error of 0.6 mas in the alignment of the axes at the catalogue epoch J1991.25, and 0.25 mas yr^{-1} in the rate of rotation of one system with respect to the other. Since the ICRS is defined by means of extragalactic sources, the Hipparcos and Tycho proper motions are quasi-inertial to within $\pm 0.25 \text{ mas yr}^{-1}$.

Thus, within the uncertainties quoted above, the Hipparcos and Tycho Catalogues represent the extension to optical wavelengths of the extragalactic reference system as realised by the International Celestial Reference Frame (ICRF). In the catalogues, the positions and proper motions are thus specified as referring to the ICRS.

Traditionally, astrometric catalogues have been referred to the mean equinox and equator of some standard epoch such as B1950 or J2000. This practice was in principle based on the *dynamical* definition of a reference system in terms of two fundamental planes, the mean equatorial plane (given by the Earth’s rotation after elimination of nutation terms) and the ecliptic plane. The intersection of the two planes defines the mean equinox as the origin of right ascension. The two planes are not inertially fixed, but move slowly due to luni-solar and planetary precession. In order to define a fixed (inertial) reference system it was necessary to specify a certain date E at which to ‘freeze’ the fundamental planes, and hence set the reference axes

for the celestial coordinates. The latter are then said to refer to the mean equinox and equator of E (say, J2000). Transformation of coordinates between reference systems of different equinoxes E_1 , E_2 is equivalent to correcting the positions for the accumulated precession between the two dates E_1 and E_2 . This process is dependent on a dynamical theory of precession and the observationally determined constants of the theory.

With the adoption of the ICRS and a purely conventional definition of celestial coordinates (through a list of adopted source positions), the mean equatorial plane and the ecliptic have lost their fundamental significance for the celestial reference system. Consequently it is not meaningful to speak of the 'equator' or 'equinox' of the ICRS, except possibly as (inappropriate) designations of the origins of α and δ in that system. In particular, no specific date is associated with the origin of α in ICRS, although it is close to the mean dynamical equinox of J2000.

The positions and proper motions in the Hipparcos Catalogue define a reference frame which is likely to be accurate, on a global scale, to about 0.1 mas (at J1991.25) and 0.1 mas yr⁻¹. Due to the large number of stars, the potential level of definition might even be a factor 10 better than these numbers, although current techniques do not allow any assessment on that accuracy level. Intrinsically, therefore, the Hipparcos Reference Frame is considerably better defined than its link to the extragalactic ICRS. Once fixed through the Hipparcos Catalogue, the Hipparcos Reference Frame will have its own independent existence as one particular realisation of the ICRS; for many years, it is likely to remain the best realisation of the ICRS in the optical domain.

1.2.3. Time Scales

The time scale used in the Hipparcos and Tycho Catalogues (for both the astrometric and the photometric data) is Terrestrial Time (TT). The practical realisation of this scale is through International Atomic Time (TAI). The basic unit of TAI and TT is the SI second, and the offset between them is conventionally 32.184 s (with deviations, attributable to the physical defects of atomic time standards, probably between the limits $\pm 10 \mu\text{s}$), so that the realisation of TT in terms of TAI is taken to be:

$$\text{TT}(\text{TAI}) = \text{TAI} + 32.184 \text{ s} \quad [1.2.1]$$

For details of the choice and definition of time scales, see the *Explanatory Supplement to the Astronomical Almanac*, 1992, University Science Books, P.K. Seidelmann (ed.).

Relativistic effects and relativistic timescales: The rate of an atomic clock depends on the gravitational potential and its motion with respect to other clocks; thus the timescale entering the equations of motion (and its relationship with TAI) depends on the coordinate system to which the equations refer. Since 1984 *The Astronomical Almanac* referred to two such timescales: Terrestrial Dynamical Time (TDT) used for geocentric ephemerides, and Barycentric Dynamical Time (TDB) used for ephemerides referred to the solar system barycentre. TDT differs from TAI by a constant offset, which was chosen to give continuity with ephemeris time. TDB and TDT differ by small periodic terms (arising from the transverse Doppler effect and gravitational red-shift experienced by the observer) that depend on the form of the relativistic theory being used: the difference includes an annual sinusoidal term of approximately 1.66 ms amplitude, planetary terms contributing up to about 20 μs , and lunar and diurnal terms contributing up to about 2 μs .

In 1991 the IAU adopted resolutions introducing new timescales which all have units of measurement consistent with the unit of time, the SI second. Terrestrial Time (TT) is the time reference for apparent geocentric ephemerides, and can be considered as equivalent to TDT. Barycentric Coordinate Time (TCB) is the coordinate time for a coordinate system with origin at the solar system barycentre. Because of relativistic transformations TDB, and therefore TT, differ in rate from TCB by approximately 49 seconds per century.

The relationships between TT, TAI and UTC: For civil and legal purposes it is necessary to have a time scale which approximates the diurnal rotation of the Earth relative to the Sun. Historically this has been known as Universal Time, but because the Earth's angular spin rate is variable, the Universal Time scale is non-uniform with respect to TAI. The civil time scale, which has been available through broadcast time signals since 1972, is known as Co-ordinated Universal Time (UTC), and differs from TAI by an integer number of seconds—it is adjusted, when judged necessary by the International Earth Rotation Service (IERS), by adding a 'leap' second at midnight on December 31, or on June 30. The transformations between the various time scales during the Hipparcos mission are given in Table 1.2.1.

Table 1.2.1. Relationship between time scales during the mission

Start interval (UTC)	End interval (UTC)	TAI-UTC (s)	TT-UTC (s)
Launch (1989 Aug 8)	1990 Jan 1 0 ^h	24	56.184
1990 Jan 1 0 ^h	1991 Jan 1 0 ^h	25	57.184
1991 Jan 1 0 ^h	1992 Jul 1 0 ^h	26	58.184
1992 Jul 1 0 ^h	end of observations	27	59.184

Tables giving the correspondences between TT (preceded by ephemeris time), TAI (since 1955), and UTC, can be found in the yearly reports of IERS as well as in the major astronomical almanacs.

1.2.4. Fundamental Constants

For the Hipparcos reductions the former IAU (1976) system of astronomical constants has been adopted, with the exception of certain planetary masses entering the Earth ephemeris. This affects primarily the position and velocity of the solar system barycentre relative to the Sun, which enter, respectively, in the computation of barycentric directions and stellar aberration.

The ephemeris for the Earth used formulae provided by the Bureau des Longitudes based on VSOP 82 (P. Bretagnon, 1982, *Astron. Astrophys.*, 114, 278) and ELP 2000 (M. Chapront-Touzé & J. Chapront, 1983, *Astron. Astrophys.*, 124, 50). Although there are small differences in certain of the constants used by the Bureau des Longitudes ephemeris and the JPL ephemeris DE200 (for example in the mass of Pluto and in the obliquity of the ecliptic) VSOP 82 was fitted to DE200 rather than directly to observations, so that the basis for the Earth ephemeris is essentially DE200. Differences between the respective ephemerides are of the order of a few mm/s in velocity, and correspondingly small in position, differences which are entirely negligible (< 0.01 mas) in the calculation of stellar aberration and parallax.

The numerical values or sources of data used in the data reductions are given in Table 1.2.2. The unit of time is the SI second, or the Julian Year when this is more convenient. The speed of light enters mainly in the computation of stellar aberration. The astronomical unit is the basis for the definition of parallax. The heliocentric and geocentric gravitational constants are used to compute the gravitational light bending by the Sun and the Earth. The obliquity of the ecliptic has no direct significance for the data reductions, but is used as a conventional value to transform between the equatorial and ecliptic systems. The Earth ephemeris is relevant for the calculation of aberration and parallax.

Table 1.2.2. Physical and astronomical constants used for the data reductions

Symbol	Meaning/Application	Value
	Unit of time	SI second as realised on the geoid
	Time scale	Terrestrial Time (TT)
Julian Year	Proper motion unit (mas yr ⁻¹)	365.25 × 86 400 s (exactly)
<i>c</i>	Speed of light	299 792 458 m s ⁻¹ (exactly)
<i>A</i>	Astronomical unit	(499.004 782 s) × <i>c</i> (exactly) = 1.495 978 701 × 10 ¹¹ m (<i>A_m</i>) = 1000 mas pc (<i>A_p</i>) = 4.740 470 446 km yr s ⁻¹ (<i>A_v</i>) = 9.777 922 181 × 10 ⁸ mas km yr s ⁻¹ (<i>A_z</i>)
<i>GS</i>	Heliocentric gravitational constant	1.327 124 38 × 10 ²⁰ m ³ s ⁻²
<i>GE</i>	Geocentric gravitational constant	3.986 005 × 10 ¹⁴ m ³ s ⁻²
<i>ε</i>	Obliquity of ecliptic (J2000.0)	23° 26' 21.448'' (exactly) = 23° 439 291 111 1 ...
	Earth ephemeris	VSOP 82/ELP 2000

Note: The astronomical unit *A* appears in all formulae relating linear measures to the parallax. Depending on the context and the units used, it is represented by a variety of numerical values, as indicated in the table. The appropriate numerical value in a given formula can always be ascertained from the units, but for clarity the specific designations *A_m*, *A_p*, *A_v* or *A_z* may also be used.

1.2.5. Conventions for Angular Coordinates

As explained in Section 1.2.2, astrometric data in the Hipparcos and Tycho Catalogues are referred to the International Celestial Reference System (ICRS), and are given for the mean catalogue epoch $T_0 = \text{J1991.25(TT)}$. With the choice of the ICRS, the angular coordinates to be used are the equatorial ones: right ascension (α) and declination (δ). Note, however, that the observations and, as a consequence, the error properties of the resulting catalogue have a strong dependence on, and a strong symmetry when expressed in, ecliptic latitude.

The proper motion is the rate of change of the position. As such it may be defined as the time derivative of the positional coordinates, $\mu_\alpha \equiv d\alpha/dt$ and $\mu_\delta \equiv d\delta/dt$. However, for the proper motion component in right ascension, the quantity tabulated in the catalogues is $\mu_{\alpha*} \equiv (d\alpha/dt) \cos \delta$. The factor $\cos \delta$ (signified by the asterisk in $\mu_{\alpha*}$) converts the proper motion in α to great-circle measure, making it directly comparable to μ_δ and to the proper motion components of other stars. For instance, the total proper motion (length of the proper motion vector) is given by $\mu = (\mu_{\alpha*}^2 + \mu_\delta^2)^{1/2}$, and the position angle θ of the proper motion vector is defined through $\mu_{\alpha*} = \mu \sin \theta$, $\mu_\delta = \mu \cos \theta$.

The standard errors in position are similarly expressed in great-circle measure: $\sigma_{\alpha*} \equiv (\text{standard error in } \alpha) \times \cos \delta = \sigma_\alpha \cos \delta$, $\sigma_\delta \equiv (\text{standard error in } \delta)$. As a result, the quoted error is the true positional uncertainty on the sky; the resulting figures show no systematic increase with increasing declination. Thus, for a star with $\delta = 80^\circ$, a typical error in right ascension is $\sigma_{\alpha*} = 1$ milliarcsec, while $\sigma_\alpha = 6$ milliarcsec.

The standard errors in proper motion refer to the uncertainties in the great-circle measures $\mu_{\alpha*}$ and μ_δ ; they are designated $\sigma_{\mu_{\alpha*}}$ and σ_{μ_δ} .

For a more rigorous definition of the standard errors, and especially for their propagation to other epochs, it is convenient to introduce local rectangular coordinates (ξ , η) in the vicinity of the star; see Section 1.2.9.

In stellar catalogues, right ascension and declination have conventionally been given in sexagesimal units, with the right ascension angle expressed as time (i.e. h m s for α and $\pm^\circ ' ''$ for δ). This is widely recognised as inconvenient for computation, and the publication of the Hipparcos and Tycho Catalogues offered an ideal opportunity to establish an alternative practice. The options for the choice of angular unit included the radian, degree, and arcsec. The choice must also consider the related units for the proper motions, the parallax, and the standard errors. It has been decided to use degrees, and decimal parts of the degree, for all celestial coordinates, and milliarcsec (1 mas = 0.001 arcsec) for the standard errors, parallax and annual proper motions.

It would clearly have been computationally convenient to express all astrometric parameters in the same units, particularly for applications involving updating of positions and for the computation of covariances. Since the use of the arcsec for parallax and annual proper motion components is convenient and well-established, and since the standard errors of the astrometric parameters are generally small numbers of milliarcsec, the milliarcsec has been adopted as the unit for all astrometric errors, as well as for parallaxes and annual proper motions. The contrasting use of (decimal) degrees for positions was considered as an acceptable compromise. While the consistent use of radians, nanoradians, or milliarcsec might have been considered as scientifically more desirable, such deviation from accepted astronomical practice was not considered to be justifiable.

Approximate right ascension and declination are nevertheless given for each object, for identification purposes, in the conventional sexagesimal units (h m s, $\pm^\circ ' ''$). By including these sexagesimal positions with truncated precision, possible confusion as to which quantities constitute the definitive Hipparcos and Tycho Catalogue positions should be avoided.

The astrometric parameters, as well as the other observational quantities in the catalogues, are given with a sufficient number of decimals to ensure that the rounding errors introduced by the finite number of decimals are much smaller than the observational uncertainties, even for the most accurate data. As a result, less accurate data may be given with several non-significant decimals. The right ascension, for example, is always given to eight decimal places (corresponding to a maximum rounding error of $0.018 \cos \delta$ mas), although fewer decimals would in principle suffice near the poles or for the less precise positions. For many purposes, especially statistical and graphical uses of the data, the rather high numerical resolution is a distinct advantage.

1.2.6. Conventions for Epochs

Time enters in the Hipparcos and Tycho Catalogues at several levels: for the standard epoch of the coordinate system, for the epochs of the individual astrometric observations; for the epoch of the published reference positions; for the unit of time for the proper motions; for the epochs, and sometimes the time dependence, in the description of double and multiple stars; and for the timing of the Hipparcos and Tycho photometric observations. For all except the photometric observations, the practical unit is the Julian year (of exactly 365.25 days or 31 557 600 s). For the propagation of stellar astrometric positions to different epochs the subtleties of the precise time-scales employed may, generally, be ignored.

This is not the case for the timing of individual photometric observations which are provided in the Hipparcos and Tycho Epoch Photometry Annexes. An internal precision of 0.01 s is achieved, although preserving such precision is not warranted in view of the

uncertainty of the photometric measurements. The observation time is given in the catalogues to 10^{-5} day, or to about one second. At this level, the precise time scale has to be specified. Adopting the TT scale for the photometric, as well as astrometric, results means that the epoch photometry is also presented on a continuous time scale (i.e. without the timing discontinuities present in UTC). Users of the epoch photometry must note the potentially significant difference between the TT and UTC time scales.

Epochs relevant to astrometry in the Hipparcos and Tycho Catalogues are expressed in years and decimal fractions of a year. Observations of solar system objects are given in terms of Julian Date (JD). The epochs in the Hipparcos and Tycho Epoch Photometry Annexes (the compilations of epoch photometry) have been corrected to the solar system barycentre by the application of light-time corresponding to the projected coordinate distance from the satellite to the barycentre, and are expressed in terms of Barycentric Julian Date, BJD(TT). This accords with the procedure widely adopted by photometrists to avoid yearly phasing of light curves. (It should be noted that this is not the same as expressing the observation epochs in barycentric coordinate time, TCB.) An offset of 2 440 000.0 days has been introduced in order to conserve space, again following a widely used convention. All Julian Dates in the Hipparcos context are in the time-scale TT, and denoted JD(TT). Some confusion in the use of Julian Dates from the literature may arise because they may be given (in the literature) either in TT or UTC, without explicit specification. It should be carefully noted that Modified Julian Date (i.e. JD–2 400 000.5) is not used in the context of the Hipparcos or Tycho Catalogues.

The standard epoch of the fundamental astronomical coordinate system J2000 corresponds to JD 2 451 545.0 terrestrial time (TT), and to the calendar date 2000 January 1, 12^h = 2000 January 1.5(TT).

Epoch definitions are based on the Julian year of 365.25 days. Thus the Julian epoch Jyyy.yy corresponds to:

$$\text{JD} = 2\,451\,545.0 + (\text{yyyy.yy} - 2000.0) \times 365.25 \quad [1.2.2]$$

In particular, the adopted catalogue epoch for the Hipparcos and Tycho Catalogues is:

$$T_0 = \text{J1991.25(TT)} = \text{JD } 2\,448\,349.0625(\text{TT}) \quad [1.2.3]$$

which is a good approximation to the mean central epoch of the observations.

Note that the adopted catalogue epoch contrasts with the standard epoch J2000 used, for example, for the Hipparcos Input Catalogue compilation. This is simply a consequence of the fact that the Hipparcos and Tycho Catalogues are observational catalogues spanning the measurement interval 1989–93.

In principle, the mean observational epochs for the individual stars could have been explicitly included in the catalogue, and the associated standard errors referred to that epoch. This would have minimised the numerical values of the tabulated positional standard errors, as well as the correlations between the position and proper motion components. However, because the astrometric parameters are derived from one-dimensional measurements (scans) across an object, with an irregular distribution both in time and position angle, the mean observational epoch is in general different in the two coordinates. This non-uniqueness of the definition of the mean observational epoch and the additional complexity in handling the individual epochs have prevented the adoption of this approach. Instead, a single common catalogue epoch has been used. The complete information on the mean observational epochs is still preserved, as explained in Section 1.2.7, in the standard errors and correlations, both of which are referred to the catalogue epoch J1991.25(TT).

1.2.7. Variance-Covariance Data and Correlations

Although the catalogue gives the first positional coordinate as right ascension, the quantity actually determined is $\Delta\alpha \cos \delta$, where $\Delta\alpha$ is a correction to an initial value, α_0 . Similarly, $\Delta\delta$ is a correction to an initial value of declination, δ_0 . The five variables, estimated by a weighted least-squares method, are therefore:

$$\begin{aligned} a_1 &= \Delta\alpha^* \equiv \Delta\alpha \cos \delta \\ a_2 &= \Delta\delta \\ a_3 &= \pi \\ a_4 &= \mu_{\alpha^*} \equiv \mu_{\alpha} \cos \delta \\ a_5 &= \mu_{\delta} \end{aligned} \tag{1.2.4}$$

In the context of least-squares estimation, all information on the quality of the estimated values a_i^{est} is contained in the 5×5 variance-covariance matrix with elements $c_{ij} = E[(a_i^{\text{est}} - a_i^{\text{true}})(a_j^{\text{est}} - a_j^{\text{true}})] (= c_{ji})$; $i, j = 1, \dots, 5$. However, the physical significance of the covariances is made more transparent by giving separately the standard errors, σ_i , of the estimated parameters and the correlation coefficients, ρ_{ij} ($= \rho_{ji}$). The elements of the variance-covariance matrix can be reconstructed from the standard errors and correlations by:

$$c_{ii} = \sigma_i^2, \quad c_{ij} = \rho_{ij} \sigma_i \sigma_j \tag{1.2.5}$$

In the catalogues, the correlation coefficients are given in the order $\rho_{21}, \rho_{31}, \rho_{32}, \rho_{41}, \dots, \rho_{54}$ and are designated $\rho_{\alpha^*}^{\delta}, \rho_{\alpha^*}^{\pi}, \rho_{\delta}^{\pi}, \rho_{\alpha^*}^{\mu_{\alpha^*}}, \dots, \rho_{\mu_{\alpha^*}}^{\mu_{\delta}}$. This sequence corresponds to the triangular part of the correlation matrix below the main diagonal, taken in row-wise order (or the upper-triangular part taken in column-wise order).

The use of the asterisk notation is not strictly required in the correlations, since the correlation coefficient is the same between (say) α and δ , as between $\alpha \cos \delta$ and δ (with $\cos \delta$ considered a fixed factor). Nevertheless, it has been retained for uniformity.

Correlation coefficients are consistently provided, when appropriate, throughout the astrometric parts of the Hipparcos and Tycho Catalogues. For the Hipparcos Catalogue, only the 10 correlation coefficients among the five standard astrometric parameters are given in the printed catalogue, while the machine-readable version gives the full set of correlation coefficients also for the more complex solutions of double and multiple systems. Correlations are provided in order to allow the standard errors of transformed quantities to be correctly estimated (see Section 1.5).

The correlation coefficients and standard errors also implicitly define the mean observational epoch for an individual object. This is in general different depending on which positional coordinate is considered; for instance, it is different in right ascension and declination, while still other mean epochs are found in galactic longitude and latitude. For a particular coordinate axis the mean observational epoch may be defined as the epoch of zero correlation between the components of position and proper motion in that coordinate; this epoch also minimises the standard error in the position coordinate. With F_n referring to fields H_n or T_n of the Hipparcos and Tycho Catalogues respectively, the mean observational epoch in right ascension is given by:

$$J1991.25 - \rho_{\alpha^*}^{\mu_{\alpha^*}} \times \frac{\sigma_{\alpha^*}}{\sigma_{\mu_{\alpha^*}}} = J1991.25 - F22 \times \frac{F14}{F17} \tag{1.2.6}$$

at which epoch the standard error in right ascension is:

$$\sigma_{\alpha^*} \sqrt{1 - (\rho_{\alpha^*}^{\mu_{\alpha^*}})^2} = F14 \times \sqrt{1 - F22 \times F22} \tag{1.2.7}$$

The mean epoch in declination is given by:

$$J1991.25 - \rho_{\delta}^{\mu_{\delta}} \times \frac{\sigma_{\delta}}{\sigma_{\mu_{\delta}}} = J1991.25 - F26 \times \frac{F15}{F18} \quad [1.2.8]$$

at which epoch the standard error in declination is:

$$\sigma_{\delta} \sqrt{1 - (\rho_{\delta}^{\mu_{\delta}})^2} = F15 \times \sqrt{1 - F26 \times F26} \quad [1.2.9]$$

If an ‘effective epoch’ for individual objects is required, it may be defined as the epoch minimising the sum of the variances of the two positional coordinates (this definition is invariant with respect to the choice of coordinate system); it is given by:

$$J1991.25 - \frac{\rho_{\alpha^*}^{\mu_{\alpha^*}} \sigma_{\alpha^*} \sigma_{\mu_{\alpha^*}} + \rho_{\delta}^{\mu_{\delta}} \sigma_{\delta} \sigma_{\mu_{\delta}}}{\sigma_{\mu_{\alpha^*}}^2 + \sigma_{\mu_{\delta}}^2} = J1991.25 - \frac{F22 \times F14 \times F17 + F26 \times F15 \times F18}{F17 \times F17 + F18 \times F18} \quad [1.2.10]$$

The effective epoch always falls between the epochs given by Equations 1.2.6 and 1.2.8. The standard errors in right ascension and declination at the effective epoch are slightly larger than the values given by Equations 1.2.7 and 1.2.9.

Observation epochs determined according to these precepts must be interpreted with caution for ‘non-standard’ solutions, such as certain orbital systems, or secondary components of ‘two-pointing’ entries fixed with respect to the primary (with large positional errors and small errors in proper motion) where the calculated mean epochs may even lie outside of the observation interval [1989.845, 1993.207].

1.2.8. The Standard Model of Stellar Motion

The five astrometric parameters given in the Hipparcos and Tycho Catalogues describe the instantaneous motion of the star relative to the solar system barycentre, in a plane perpendicular to the line of sight. The sixth parameter needed to completely specify the space motion is the radial velocity, normally determined by spectroscopic means. The physical model underlying this description is that the stars move through space with constant velocity vector. This section reviews the formulae needed to calculate the positions of the stars for arbitrary epochs according to this ‘standard’ model of stellar motion, which is the basis for the main astrometric reduction of the Hipparcos data.

In reality the space velocity may vary because of perturbations from a companion star, planets, or the gravity field of other stars; ultimately, the curvatures of the galactic orbits of the star and of the Sun must be taken into account when extrapolating to sufficiently distant epochs. Except for the perturbations by close companions, these effects can generally be ignored in the computation of stellar positions over time spans of a century or less, even at the precision level of Hipparcos. Uniform space motion thus remains the standard model for the majority of stars in the Hipparcos Catalogue. The perturbations by close companions, when significant, are covered by the special solutions of type G and O in the Double and Multiple Systems Annex; see Section 2.3.

The five astrometric parameters, as given in the catalogues, refer to the common epoch $T_0 = J1991.25(\text{TT})$. Since the parameters change with time, subscript $_0$ will be used, when necessary, to denote their values as given in the catalogues. In particular, the barycentric direction towards the star at the epoch T_0 is specified by the celestial coordinates α_0 , δ_0 given in Fields H8 and H9 of the Hipparcos Catalogue, or Fields T8 and T9 in the Tycho Catalogue. Equivalently, this direction may be expressed as a unit vector \mathbf{r}_0 , whose rectangular equatorial coordinates are given by:

$$\mathbf{r}_0 = \begin{pmatrix} \cos \delta_0 \cos \alpha_0 \\ \cos \delta_0 \sin \alpha_0 \\ \sin \delta_0 \end{pmatrix} \quad [1.2.11]$$

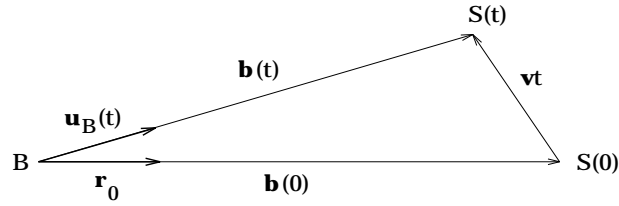


Figure 1.2.1. Definition of the barycentric coordinate direction $\mathbf{u}_B(t)$ to a star moving with uniform space velocity \mathbf{v} relative to the solar system barycentre B . $S(0)$, $S(t)$ are the positions of the star at times 0 and t .

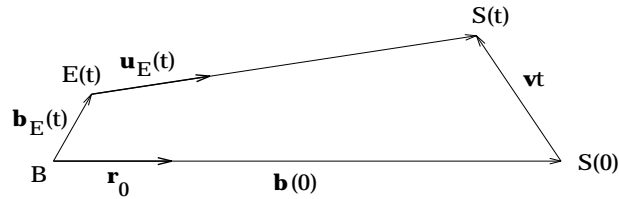


Figure 1.2.2. Definition of the topocentric coordinate direction $\mathbf{u}_E(t)$. $\mathbf{b}_E(t)$ = barycentric vector to the observer at $E(t)$; other designations as in Figure 1.2.1.

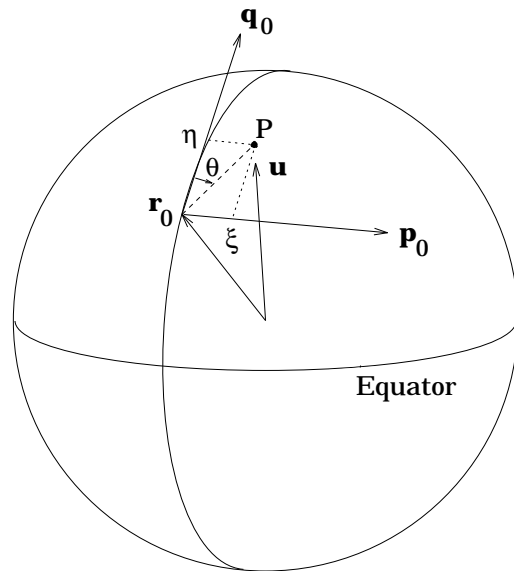


Figure 1.2.3. Definition of local plane coordinates (ξ, η) in the tangent plane at \mathbf{r}_0 . The orthogonal unit vectors \mathbf{p}_0 and \mathbf{q}_0 , which span the tangent plane, are oriented in the local directions of $+\alpha$ and $+\delta$ at \mathbf{r}_0 , respectively. P is the projection of unit vector \mathbf{u} in the tangent plane, ξ and η its rectangular coordinates with respect to \mathbf{p}_0 and \mathbf{q}_0 . The position angle θ of \mathbf{u} with respect to \mathbf{r}_0 is indicated.

The standard model is a prescription for calculating the celestial position (direction) at the arbitrary epoch $T = T_0 + t$.

Two cases are considered: the calculation of the *barycentric* direction (as seen from the solar system barycentre), and of the *topocentric* direction (as seen from an arbitrary location such as a point on the Earth). The two directions differ by the parallactic displacement of the star.

Note that all directions discussed in this section are *coordinate directions* corresponding to the coordinate differences between the source and observer in the barycentric coordinate frame. The transformation of coordinate directions to observed (proper) directions is discussed in Volume 3, Chapter 12; see also, for example, C.A. Murray, 1983, *Vectorial Astrometry*, Bristol, and the annual issues of *The Astronomical Almanac*, Section B. Furthermore, light-time effects are ignored here, i.e. no distinction is made between the time of light emission at the object and the time of light reception by the observer; both are designated T (or t) as if the speed of light were infinite. It can be shown that the errors introduced by this approximation are negligible as far as the modelling of the coordinate direction as a function of time is concerned: even in an extreme case such as Barnard's star, the errors are smaller than those caused by neglecting the curvature of its galactic orbit. See also the note after Equation 1.2.20.

The barycentric coordinate direction: In the standard model the barycentric vector to the star is given by:

$$\mathbf{b}(t) = \mathbf{b}(0) + \mathbf{v}t \quad [1.2.12]$$

where $\mathbf{b}(0) = \mathbf{r}_0 A / \pi_0$ is the barycentric position at the catalogue epoch and \mathbf{v} the constant space velocity (Figure 1.2.1). Using angular brackets to denote vector normalisation, the barycentric direction is given by the unit vector $\mathbf{u}_B(t) = \langle \mathbf{b}(0) + \mathbf{v}t \rangle$. This expression is unsuitable for calculation because of the appearance of the parallax in the denominator. A more convenient form is:

$$\mathbf{u}_B(t) = \langle \mathbf{r}_0 + \mathbf{v}(\pi_0/A)t \rangle \quad [1.2.13]$$

where the normalisation factor is very close to unity.

The proper motion vector is the time derivative of the unit vector $\mathbf{u}_B(t)$. From its equatorial components at epoch T_0 , $\mu_{\alpha*0}$ and $\mu_{\delta 0}$, the proper motion vector is calculated as:

$$\boldsymbol{\mu}_0 = \mathbf{p}_0 \mu_{\alpha*0} + \mathbf{q}_0 \mu_{\delta 0} \quad [1.2.14]$$

where \mathbf{p}_0 and \mathbf{q}_0 are the unit vectors in the directions of increasing α and δ at \mathbf{r}_0 (see Figure 1.2.3):

$$\mathbf{p}_0 = \begin{pmatrix} -\sin \alpha_0 \\ \cos \alpha_0 \\ 0 \end{pmatrix} \quad \mathbf{q}_0 = \begin{pmatrix} -\sin \delta_0 \cos \alpha_0 \\ -\sin \delta_0 \sin \alpha_0 \\ \cos \delta_0 \end{pmatrix} \quad [1.2.15]$$

The orthogonal vector triad $[\mathbf{p}_0 \ \mathbf{q}_0 \ \mathbf{r}_0]$ is known as the *normal triad* at \mathbf{r}_0 relative to the equatorial system, and is important for the definition of differential quantities such as the proper motion components, the standard errors in position, or the local plane coordinates defined in Section 1.2.9. The normal triad should be regarded as a fixed set of coordinate axes completely defined by the given values of α_0 and δ_0 ; it is thus unaffected, for instance, by the observational uncertainty of these angles.

The tangential vectors at \mathbf{r}_0 relative to the equatorial system can be written as $\mathbf{p}_0 = \langle \mathbf{z} \times \mathbf{r}_0 \rangle$ and $\mathbf{q}_0 = \mathbf{r}_0 \times \mathbf{p}_0$, where \mathbf{z} is equatorial polar direction (towards $\delta = +90^\circ$). In equatorial rectangular coordinates, where $\mathbf{z} = (0 \ 0 \ 1)'$ and \mathbf{r}_0 is given by Equation 1.2.11, evaluation of these expressions gives Equation 1.2.15. Other

coordinate systems, e.g. the ecliptic and galactic systems, have different normal triads at \mathbf{r}_0 corresponding to their polar directions (see Section 1.5.3).

Writing the space velocity as $\mathbf{v} = \boldsymbol{\mu}_0 A / \pi_0 + \mathbf{r}_0 V_{R0}$, Equation 1.2.13 becomes:

$$\mathbf{u}_B(t) = \langle \mathbf{r}_0 + (\mathbf{p}_0 \mu_{\alpha*0} + \mathbf{q}_0 \mu_{\delta 0} + \mathbf{r}_0 \zeta_0) t \rangle = \langle \mathbf{r}_0 (1 + \zeta_0 t) + \boldsymbol{\mu}_0 t \rangle \quad [1.2.16]$$

where:

$$\zeta_0 = V_{R0} \pi_0 / A \quad [1.2.17]$$

is the relative change in distance per year.

It is seen from Equation 1.2.16 that $\mu_{\alpha*0}$, $\mu_{\delta 0}$ and ζ_0 are the components of the space velocity, scaled by the inverse distance at epoch T_0 , along the vectors of the normal triad at \mathbf{r}_0 . The quantity ζ_0 is therefore the equivalent, in the radial direction, to the proper motion in the tangential direction, and is statistically of a similar size when expressed in the corresponding unit. With V_{R0} expressed in $[\text{km s}^{-1}]$, π_0 in $[\text{mas}]$, and A in $[\text{km yr s}^{-1}]$ ($= A_V$ in Table 1.2.2), Equation 1.2.17 gives ζ_0 in $[\text{mas yr}^{-1}]$. Using A expressed in $[\text{mas km yr s}^{-1}]$ ($= A_Z$ in Table 1.2.2) gives ζ_0 in $[\text{yr}^{-1}]$, appropriate for calculating the factor $(1 + \zeta_0 t)$ in Equation 1.2.16.

The topocentric coordinate direction: The coordinate difference between the object and the observer is given by the vector $\mathbf{b}(0) + \mathbf{v}t - \mathbf{b}_E(t)$, where \mathbf{b}_E is the barycentric position of the observer (Figure 1.2.2). The topocentric direction is therefore given by the unit vector:

$$\mathbf{u}_E(t) = \langle \mathbf{r}_0 (1 + \zeta_0 t) + \boldsymbol{\mu}_0 t - \mathbf{b}_E(t) \pi_0 / A \rangle \quad [1.2.18]$$

where, again, the normalising factor is very close to unity.

For Earth-bound applications of the Hipparcos and Tycho Catalogues it is sufficient to consider a geocentric observer. In that case $\mathbf{b}_E(t)/A$ may be taken directly from a barycentric equatorial ephemeris of the Earth, such as given in Section B of *The Astronomical Almanac*.

Equations 1.2.14 to 1.2.17 allow the barycentric direction to be calculated in terms of the six parameters α_0 , δ_0 , π_0 , $\mu_{\alpha*0}$, $\mu_{\delta 0}$ and V_{R0} . The topocentric direction is similarly obtained by using Equation 1.2.18 instead of Equation 1.2.16. Either direction may be converted to celestial coordinates $\alpha(t)$, $\delta(t)$ using the component form of the unit vector:

$$\mathbf{u}(t) = \begin{pmatrix} \cos \delta(t) \cos \alpha(t) \\ \cos \delta(t) \sin \alpha(t) \\ \sin \delta(t) \end{pmatrix} \quad [1.2.19]$$

It should be noted that many applications of the catalogue data, such as the further transformation into ‘proper’ directions for comparison with observations, is best done using vector calculus, in which case $\alpha(t)$, $\delta(t)$ are usually never explicitly needed.

Secular or perspective acceleration: The ζ_0 introduced by Equation 1.2.17 is the relative change in distance per unit time. The factor $(1 + \zeta_0 t)$ appearing in the normalising factors of Equations 1.2.16 and 1.2.18 thus accounts for the perspective diminishing of the proper motion of a receding object. This is known as the ‘secular’ or ‘perspective’ acceleration of the star. It is important to note that it is a purely geometrical effect which does not correspond to any physical action on the star.

Table 1.2.3. Radial velocities used for the correction of perspective acceleration

HIP	V_R (km s ⁻¹)	HIP	V_R (km s ⁻¹)
439	+22.9	71681	-18.1
3829	-38.0	71683	-26.2
5336	-98.1	74234	+308.0
15510	+86.7	74235	+294.3
19849	-42.7	86990	-115.0
24186	+245.5	87937	-111.0
26857	+105.6	99461	-129.8
54035	-84.3	104214	-64.8
54211	+68.8	104217	-64.3
57939	-99.1	108870	-40.4
70890	-16.0		

The radial velocity of HIP 3829 has been adjusted from the observed value of +15 km s⁻¹ to account for gravitational redshift and pressure shift (note that the value used for the data reductions is significantly different from the value given in the Hipparcos Input Catalogue).

The accumulated effect on the position is approximately given by $\zeta_0 \mu_0 t^2$. It is thus proportional to $\pi \times \mu \times V_R$ and practically negligible except for some nearby, high proper motion stars with large radial velocity. For the purpose of the Hipparcos data reductions, available ground-based radial velocity estimates for particularly critical objects were used as given in Table 1.2.3. In all other cases, V_R was assumed to be zero. The sample in Table 1.2.3 contains all Hipparcos stars with available radial velocities, for which the predicted accumulated effect over two years exceeds 0.1 mas.

Computation of transverse motions: The component of the stellar space velocity tangent to the line of sight, V_T , is traditionally calculated as:

$$V_T = \frac{A_v \mu}{\pi} \quad [1.2.20]$$

where $A_v = 4.74047\dots$ equals the astronomical unit, expressed in [km yr s⁻¹] (see Table 1.2.2). Many textbooks give slightly different values for A_v , usually because they assume the (now obsolete) tropical year as time unit for the proper motions.

Equation 1.2.20 is consistent with the standard model of stellar motions, in that it neglects the light-time effects. As mentioned before, this is an acceptable approximation for the modelling of the apparent motions of stars over historical time spans. However, light-time effects are not necessarily negligible for the calculation of the transverse velocity. In principle, the space velocity should be defined as $\mathbf{v} = d\mathbf{b}/dt^*$, where $t^* = t - b/c$ is the time of light emission at the star. Since $dt^*/dt = 1 - V_R/c$, it follows that the correct expression for the space velocity, in terms of the observed proper motion and radial velocity, is:

$$\mathbf{v} = (\boldsymbol{\mu} A/\pi + \mathbf{r}_0 V_R)(1 - V_R/c)^{-1} \quad [1.2.21]$$

The expression for V_T , Equation 1.2.20, should thus in principle be amended to include the Doppler factor $k = (1 - V_R/c)^{-1}$. While the omission of this factor has drastic consequences for the interpretation of relativistic motions (for example, as superluminal expansion), it causes only a very small error on the calculated space velocities of galactic stars and there is generally no harm in using Equation 1.2.20 as it stands. (Equation 1.2.21 assumes that V_R is defined as the rate of change in coordinate distance per unit of t . Depending on the formula used to convert the observed redshift to a velocity, this may or may not be true.) The calculation of space coordinates and velocities is more extensively treated in Section 1.5.6.

1.2.9. Use of Local Plane Coordinates

While the use of vector calculus is generally to be recommended for rigorous computation of celestial directions, there are circumstances where small changes in the directions are best described differentially with respect to some reference direction. A typical example is the use of separation and position angle to characterise the relative positions of the components in a double or multiple star, or the definition of orbital parameters for binaries.

The classical method to express such positional offsets is by means of direct differences in the celestial coordinates. In the equatorial system these are typically denoted $\Delta\alpha \cos \delta$ and $\Delta\delta$, where the $\cos \delta$ factor is introduced to make the arc measures comparable in the two coordinates.

Quantities like $\Delta\alpha \cos \delta$ and $\Delta\delta$, while perfectly serviceable in many situations, suffer from a certain ambiguity or lack of rigour which make them unsuitable for general computation. Unpleasant or unforeseen things could happen especially when used for stars close to the celestial poles. These disadvantages can be eliminated by adopting a precise definition of offset coordinates. Among several possible choices, the one adopted presently appears eminently suitable for propagating positions according to the standard model described in the previous section. It corresponds closely to the use of ‘standard coordinates’ in photographic astrometry. In order to avoid association with the less well-defined $\Delta\alpha \cos \delta$ and $\Delta\delta$, a specific notation (ξ, η) is introduced for the offset coordinates according to this definition.

The offset coordinates (ξ, η) are defined as rectangular coordinates in the tangent plane at the fixed reference point \mathbf{r}_0 , with ξ and η increasing respectively in the directions of the vectors \mathbf{p}_0 and \mathbf{q}_0 (Figure 1.2.3). The normal triad $[\mathbf{p}_0 \mathbf{q}_0 \mathbf{r}_0]$, and hence the offset coordinate system, is completely defined by the barycentric celestial coordinates α_0 and δ_0 at the catalogue epoch T_0 ; its equatorial components are given by Equations 1.2.11 and 1.2.15.

For the arbitrary direction \mathbf{u} the offset coordinates are calculated as:

$$\begin{aligned}\xi &= \frac{\mathbf{p}'_0 \mathbf{u}}{\mathbf{r}'_0 \mathbf{u}} \\ \eta &= \frac{\mathbf{q}'_0 \mathbf{u}}{\mathbf{r}'_0 \mathbf{u}}\end{aligned}\tag{1.2.22}$$

where the prime (') denotes scalar product, or the transpose of the column matrix representing the vector. Note that Equation 1.2.22 does not require that \mathbf{u} is of unit length. Conversely, the direction may be calculated from the offset coordinates as:

$$\mathbf{u} = \langle \mathbf{r}_0 + \mathbf{p}_0 \xi + \mathbf{q}_0 \eta \rangle\tag{1.2.23}$$

An alternative way to express the offset position is by means of polar coordinates ϱ (separation) and θ (position angle), as used for instance for double stars. The exact relation between the two representations is given by:

$$\begin{aligned}\xi &= \tan \varrho \sin \theta \\ \eta &= \tan \varrho \cos \theta\end{aligned}\tag{1.2.24}$$

Inserting Equation 1.2.16 in 1.2.22 gives a particularly simple expression for the offset coordinates of the barycentric position according to the standard model:

$$\begin{aligned}\xi_B(t) &= \frac{\mu_{\alpha*0}t}{1 + \zeta_0 t} \\ \eta_B(t) &= \frac{\mu_{\delta 0}t}{1 + \zeta_0 t}\end{aligned}\tag{1.2.25}$$

As seen from the barycentre, the star moves in a straight line in the (ξ, η) plane. To the extent that $\zeta_0 t$ can be neglected the motion is, moreover, linear in time. For the topocentric direction the offset coordinates become:

$$\begin{aligned}\xi_E(t) &= \frac{\mu_{\alpha*0}t - \mathbf{p}'_0 \mathbf{b}_E(t) \pi_0 / A}{1 + \zeta_0 t - \mathbf{r}'_0 \mathbf{b}_E(t) \pi_0 / A} \\ \eta_E(t) &= \frac{\mu_{\delta 0}t - \mathbf{q}'_0 \mathbf{b}_E(t) \pi_0 / A}{1 + \zeta_0 t - \mathbf{r}'_0 \mathbf{b}_E(t) \pi_0 / A}\end{aligned}\tag{1.2.26}$$

To the accuracy of the Hipparcos data the last term in the denominators of Equation 1.2.26 can always be ignored.

In Parts G, O and V of the Double and Multiple Systems Annex (Section 2.3) the non-linear motions of the photocentres of unresolved binaries are expressed relative to a fiducial point moving according to the standard model in Section 1.2.8, with astrometric parameters as given in the main catalogue (and assuming $\zeta_0 = 0$ except for the stars in Table 1.2.3). See Equations 2.3.1, 2.3.5, and 2.3.9. The offset coordinates of the fiducial point are denoted $\xi_s(t)$ and $\eta_s(t)$ and may be calculated from Equation 1.2.25 or 1.2.26 depending on whether barycentric or topocentric positions are required.

